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OPTIMAL PUBLIC INFORMATION DISSEMINATION.

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Introducing Multiplier Effects into a Generalized Beauty Contest

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Abstract

We develop a two-period generalized beauty contest to study the optimal level of publicity when disclosed information is subject to multiplier effects. We build upon the static case, where all agents receive a private signal about an unknown fundamental state and only a fraction of all agents receive an additional public signal. However, in our model, agents no longer act simultaneously; rather, agents informed by both signals act in the first period, while those uninformed act in the second period and learn about the public signal through a multiplier signal. We show that in the unique equilibrium of our sequential game, informed agents overreact more strongly to public signals. The optimal dissemination of public information is thus considerably lower than the static case suggests. Multiplier effects might decrease overall welfare if coordination incentives are sufficiently strong. Our results hold relevance for the optimal information policy design of public authorities.

Keywords: generalized beauty contest, monetary policy, optimal communication, strategic complementarities

JEL: D82, D83, E5

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Public authorities such as central banks have a profound impact on financial markets as agents pay close attention to information disclosed. Public information serves agents as a valuable signal for an economy's fundamentals and - by influencing market expectations - it affects asset prices and interest rates. However, Morris and Shin (2002) (henceforth MS) have shown in their seminal generalized beauty contest model that the dissemination of public information might also have detrimental welfare effects due to agents' overreaction in markets characterized by strategic complementarities. Accordingly, it might be optimal for central banks to strategically withhold some information. Cornand and Heinemann (2008) (henceforth CH) introduce the degree of publicity as an efficient instrument to account for overreactions to public information. Although multiplier effects fostering the dissemination of public information are acknowledged concerning their relevance to assess the optimal degree of publicity, such distortions to a straightforward management of publicity and overall market expectations have not been modeled. For instance, consider the media that picks up, interprets and spreads disclosed information or the process of social learning, where initially uninformed agents observe actions of market participants and infer underlying information. These dynamics in the penetration of public information substantially alter central banks' optimal policy design.

In this paper, we extend the analysis presented by CH to a two-period model incorporating multiplier effects into the generalized beauty contest with strategic complementarities. In our framework, agents who are uninformed about the public signal learn from a correlated multiplier signal in a fashion presented by Myatt and Wallace (2014). Similar to herding models with pay-off externalities (cp. for instance Dasgupta 2000 or Chari and Kehoe 2004), agents have the incentive to coordinate within and across periods. This extension from the static to a sequential setting allows us to analyze the interplay of pay-off externalities induced by strategic complementarities and information externalities inherent to multiplier effects such as social learning. We show that multiplier effects might decrease overall welfare if coordination incentives are sufficiently strong. However, the overreaction to disclosed public information will exceed the overreaction in the static case. This finding results from informed agents anticipating the behavior of the uninformed agents, who aim to coordinate across periods by relying on the multiplier signal. In response to multiplier effects spreading disclosed information, the central bank optimally chooses a lower level of publicity ex ante and thus excludes more agents from receiving the public information. Furthermore, the central bank discloses information at maximal precision if the respective public signal is not extremely imprecise.

The relevance of public information policy has grown considerably since the mid-1990s, when central bank actions and communication became increasingly transparent. This increase in transparency was justified by two main arguments: first, it had become widely accepted that increased transparency improves the credibility of the democratic legitimacy of the central bank actions; and second, increased transparency was expected to improve the effectiveness of monetary policy by enabling central banks to more successfully manage market expectations (Woodford 2003; Haan et al. 2007).

Following the generalized beauty contest model introduced by MS, a broad discussion has emerged, questioning the superiority of comprehensive central bank transparency. Financial markets are predominantly characterized by strategic complementarities, which lead to agents striving to match the economy's fundamentals and - at the same time - the likely actions of other market participants. Therefore, public information contains useful information about both the fundamentals and the likely actions of other market participants, merely given its public nature. This dichotomous informational value allows agents to more effectively coordinate with the actions of others by placing a disproportionally high weight on public information. Markets are subsequently characterized by an overreaction to public information and they fail to establish the socially desirable outcome of weights being assigned according to signals' relative precision. Consequently, it might be optimal for a central bank to reduce transparency to avoid such an overreaction to public information.

As Woodford (2005) shows, this undesirable outcome of public information dissemination crucially depends on the specification of the welfare function: if coordination per se is a welfare objective, MS's proposition fails to hold. Svensson (2006) argues that the result established by MS only holds for an implausibly low precision of public information relative to private information. Morris et al. (2006) replied that assuming correlated signals, the result even holds when public information is more precise than private information.

A number of theoretical papers have considered partial publicity as a policy instrument to avoid the detrimental welfare effects of public information dissemination under strategic complementarities. Morris and Shin (2007) consider one public and n-semipublic signals within a modified generalized beauty contest game. In equilibrium, the weight accorded to the semipublic signals decreases with increasing fragmentation and there is a trade-off between fragmentation and the precisions of signals. CH emphasize that the MS framework only offers the possibility to reduce the precision of public information to mitigate the potentially detrimental effects of disseminating public information. In relaxing this assumption, they introduce an additional parameter allowing the central bank to restrict the fraction of market participants receiving the public information and thus actively manage the publicity of announcements as a strategic means to withhold information. They show that it can be optimal to disseminate public information with maximum precision to a certain fraction of all agents only.[3]

Arato et al. (2014) introduce endogenous information acquisition to the framework of CH. In their model, public information is costly and agents can choose whether or not to gather the public information. As in Hellwig and Veldkamp (2009) - where agents' actions are strategic complements - information acquisition becomes a strategic complement, leading to multiple equilibria. Arato et al. show that the central bank can guide agents to an unique partial publicity equilibrium if an increasing pricing rule is introduced, rather than using constant

[3] Trabelsi (2010) emphasizes the similarity of the fragmentation approach in Morris and Shin (2007) and the publicity approach in CH. By introducing private signals in addition to the n-semipublic signals in the framework of Morris and Shin, Trabelsi shows that both approaches induce the same impact of reducing the detrimental effect of public information: one might either reduce the publicity of public information or introduce fragmented information.

prices for public information.[4]

Another recent paper related to our research objective is Molavi et al. (2014). Contrary to our game, agents do not receive an exogenous public signal but rather observe some neighboring agents' actions. By studying social learning in a dynamic generalized beauty contest game, they show that in a Markov perfect Bayesian equilibrium agents eventually reach consensus in their actions. However, Molavi et al. focus on the asymptotic properties of actions due to social learning in a generalized beauty contest framework, while we are interested in the optimal level of publicity of the public information when a coordination motive and social learning as one potential multiplier effect interact.

The two-period model developed in Angeletos and Werning (2006) resembles our framework with strategic complementarities. Similar to the framework in CH, agents are divided into two groups: the first group merely receives a private signal - which is an unbiased signal of the underlying state - and subsequently acts in the first period, whereby average actions reflect the fundamental; and the second group acts in the second period and receives a private signal as well as a noisy public signal about the average action of the first period. This endogenous public signal serves as a focal point for agents in the second period. The authors show that such an endogenously modelled public signal might lead to a multiplicity of equilibria rather than a unique solution. However, agents have a binary action set of attacking or not-attacking the status-quo while we study the equilibrium for a continuous action set.

Contributing to the outlined literature, our results hold relevance for the optimal design of central bank communication. Multiplier effects transmit disclosed public information from informed to initially uninformed agents, thus reducing - ceteris paribus - the optimal degree of publicity. We show that inter-temporal coordination motives aggravate agents' overreaction to public information if multiplier signals contain substantial information. Given sufficiently strong coordination incentives, multiplier effects might decrease overall welfare.

The remainder of the paper is organized as follows. Section 1 motivates our approach to analyze the generalized beauty contest in a sequential two-period model, introduces our model and derives the equilibrium. Section 2 presents our welfare analysis. Section 3 discusses our model in context of heterogeneous private information, asymmetric media reporting and implications from related experimental studies, before Section 4 concludes.

1 A Sequential Generalized Beauty Contest Model

1.1 A two-period approach

In the static model by CH, all agents receive a private signal about an unknown fundamental state of the economy. A fraction of all agents additionally receive a public signal, while all others remain uninformed about this signal issued by the central bank. It is assumed that the central bank can determine the fraction of agents receiving its public signal by designing their public

[4]Other models introducing costly information acquisition to the generalized beauty contest framework of MS include Colombo and Femminis (2008), Ui (2014) and Colombo et al. (2014).

announcements accordingly. However, informed and uninformed agents act simultaneously and thus the model constitutes a static generalized beauty contest game. There is neither a time lag in agents' decisions nor any multiplier effects. Accordingly, the model does not account for a crucial characteristics of financial markets where disclosed information penetrates the market through reporting of the media or the observation of actions.

Our model can be rationalized as follows: agents strive to align their actions to the unknown fundamental state and - at the same time - follow a coordination motive. They all listen to public announcements of the central bank and face a signal-extraction problem of inferring the respective signal conveyed by the announcement. However, only a certain fraction are capable of inferring the public signal. In the words of Morris and Shin (2007): *"... as any central banker knows, it is not so easy to communicate information in such a way that it become(s) common knowledge within the private sector. If different listeners interpret an announcement differently, then the content of the announcement does not become common knowledge. If some listeners pay attention to the announcement, while others do not, then the content of the announcement does not become common knowledge. Intuitively, the more one attempts to communicate, the more likely it is that some listeners will not pay attention to all the information, and the less common knowledge."* All others have to wait an instant of one period to observe the actions of informed agents or to gather information provided by the media to gain insights into the newly announced public signal and learn about the likely common action in the market, as well as the fundamental state.

Similar considerations follow when the public signal is interpreted as a trigger to initiate an action in the first place. One might assume the fraction of uninformed agents that did not receive or failed to understand the public signal as having no intention to take an action, since no new information seems to be disclosed. Only through multiplier effects is a reaction induced. Hence, in this scenario, the decision is not delayed strategically to receive additional information through observational learning or reporting of the media. The delay rather follows from the lack of a trigger that initiates an action, which – to the informed agents – has been the observation of the public signal.

In our model, sequential choice and intertemporal coordination incentives interact. For instance, the game might feature information externalities due to social learning and payoff externalities due to strategic complementarities. As a result, informed agents are concerned about how their successors might act. This anticipation aspect distinguishes our model from most herding models following the seminal contributions of Banerjee (1992) and Bikhchandani et al. (1992), where the only externality is a purely informational one. With strategic complementarities being absent in many herding models, agents do not conduct forward-looking behavior, which makes the models more difficult to apply to actual financial markets (Dasgupta 2000).

Multiplier effects cause a dynamic penetration of public information and might raise publicity of disclosed information above the degree initially intended by the central bank. The focal role of the public signal continues to the second period. Moreover, as mentioned above, informed agents in our game account for their successor's subsequent play. This anticipation effect amplifies the focal role of the public signal, which alters the optimal level of publicity in comparison to the static CH case. Put simply, optimal central bank behavior has to account for decisions taken non-simultaneously, as well as the dynamics of the process by which public information is disseminated and influenced by multiplier effects.

1.2 The model

In the following, we first introduce agents' utility and the signal structure of the game, before subsequently formulating the optimal actions conditional upon available signals. Finally, we identify the unique linear equilibrium of the game.

Utility

There is a continuum of agents indexed by $i \in [0, 1]$. There are two periods indexed by $t = 0, 1$. Each agent i acts in either $t = 0$ or $t = 1$ but never in both periods and chooses an action $a_i \in \mathbb{R}$, which maximizes his utility given by

$$U_i(a, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}) \tag{1}$$

with

$$L_i \equiv \int_0^1 (a_i - a_j)^2 dj, \ \bar{L} \equiv \int_0^1 L_j dj,$$

where θ is the fundamental state drawn from the real line and r is a constant with $0 < r < 1$. According to (1), agents minimize the loss from the squared distance to the fundamental state of the current period (first term) and minimize the individual loss compared to the average loss from discoordination (second term).

Signal structure

All agents receive a private signal that is identically distributed around $\theta : x_i = \theta + \varepsilon_i \ with \ \varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. The precision of private signals is defined as $\beta \equiv \frac{1}{\sigma_\varepsilon^2}$. All private signals are uncorrelated.

A fraction P of all agents i observe the realization of a public signal $y = \theta + \eta \ with \ \eta \sim N(0, \sigma_\eta^2)$. We define $P \equiv \frac{informed \ agents}{all \ agents}$ such that $0 < P \leq 1$. Public signals are perfectly correlated, i.e. every informed agent observes exactly the same public signal with precision $\alpha \equiv \frac{1}{\sigma_\eta^2}$. For simplicity, we assume an exogenous decision order. Agents who are informed about the public signal y take an action in the first period $t = 0$, whereas uninformed agents take an action in the second period $t = 1$.[5]

[5] We might well construct a scenario in which agents face (constant) waiting costs when choosing to delay their actions or act immediately. However, agents being informed about the public signal can be assumed to have weaker incentives to wait in order to learn from multiplier signals such as observable actions than uninformed agents (cp. Frisell 2003). Depending on the significance of waiting costs, there should be an equilibrium where all informed agents act in the first period and all uninformed agents delay their action to the second period. This case would be equivalent to our scenario featuring an exogenous decision sequence. Please note that such an endogenously derived decision sequence would substantially complicate our analysis. Since our focus lies on optimal publicity given multiplier effects and not on the distinct timing of actions, we assume the decision order to be exogenous and abstain from modeling waiting costs.

Agents uninformed about the public signal y receive a noisy exogenous multiplier signal in $t = 1$, defined as

$$s_i = \theta + \eta + \varphi_i \text{ with } \varphi_i \sim N(0, \sigma_\varphi^2), \mu \equiv \frac{1}{\sigma_\varphi^2} \text{ and } \delta \equiv \frac{1}{\sigma_\eta^2 + \sigma_\varphi^2}. \tag{2}$$

Since $y = \theta + \eta$, $s_i = y + \varphi_i$. For vanishing idiosyncratic noise ($\sigma_\varphi^2 \to 0$), s_i becomes y and agents in $t = 1$ can coordinate more easily with agents from the same period as well as from $t = 0$. The (semi-) public signals y and s_i are correlated as they share the common noise term η, whereby correlation can be measured by

$$\rho = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\varphi^2}. \tag{3}$$

This modelling approach follows the generalized signal structure proposed by Myatt and Wallace (2008, 2014), which abandons the simple public-private signal taxonomy. The authors describe this specification as the *"sender-receiver noise model of communication"*.

Note that for $P = 1$, the game collapses to the static game introduced by MS as all agents are informed, act in the same period $t = 0$ and there are no multiplier effects. On the contrary, the fewer agents who receive the public signal y, the more agents who act in $t = 1$ and learn about y through s_i. Hence, agents of both periods have to account for the focal role of the multiplier signal s_i. However, the focal role of s_i increases for informed agents in the correlation with y as measured by ρ. For $\rho \to 0$ the average action in $t = 0$ and $t = 1$ become unrelated in the sense that there is no anticipation effect as the average action in $t = 1$ is merely expected to reflect the fundamental. In this case our model approaches the CH framework. Accordingly, our richer setting represents a more general framework comprising MS and CH as special cases.

Finally, it is assumed that all agents process the signals in a linear fashion - which is common knowledge - such that

$$a_{i0}(\Omega) = \gamma x_i + (1 - \gamma) y \tag{4}$$

and

$$a_{i1}(\Omega) = \gamma x_i + (1 - \gamma) s_i \tag{5}$$

with $\gamma \in [0, 1]$ being the weight on the private signal. We thus restrict our attention to linear equilibria. Following the related literature, we assume that $r, P, \alpha, \beta, \delta$ and ρ are common knowledge.

Optimal action

For both informed agents and uninformed agents, the FOC of (1) is the utility maximizing condition

$$a_{it}^* = (1 - r) E_i(\theta|\Omega) + r E_i(\bar{a}|\Omega). \tag{6}$$

However, the information set Ω to form the expectations $E_i(\theta)$ and $E_i(\bar{a})$ differs between the two groups. The fraction P of informed agents receive a private signal x_i and the public signal

y. The fraction $(1-P)$ of uninformed agents receive x_i and a multiplier signal s_i. We can formulate optimal individual actions conditional upon available signals. For informed agents, (6) becomes

$$a_{i0}^{*} = (1-r)E_i(\theta|x_i, y) + rE_i(\bar{a}|x_i, y), \tag{7}$$

where

$$E_i\left(\theta|x_i, y\right) = \frac{\beta}{\alpha+\beta}x_i + \left(1 - \frac{\beta}{\alpha+\beta}\right)y$$

and

$$E_i(\bar{a}|x_i, y) = PE_i(\bar{a}_0|x_i, y) + (1-P)E_i(\bar{a}_1|x_i, y).$$

For uninformed agents, we can write

$$a_{i1}^{*} = (1-r)E_i(\theta|x_i, s_i) + rE_i(\bar{a}|x_i, s), \tag{8}$$

where

$$E_i(\theta|x_i, s_i) = \frac{\beta}{\delta+\beta}x_i + \left(1 - \frac{\beta}{\delta+\beta}\right)s_i$$

and

$$E_i(\bar{a}|x_i, s_i) = PE_i(\bar{a}_0|x_i, s_i) + (1-P)E_i(\bar{a}_1|x_i, s_i).$$

While the expectations about the fundamental are equivalent to the static games of MS and CH - where private and public signals are weighted by their relative precision - the expectation about the average action differs, as it comprises expectations concerning the average action in both $t=0$ and $t=1$. These expectations about average actions are weighted by the fraction of agents acting in the respective period. It becomes clear that besides the signal structure, the main difference of our framework compared with previous models lies in the inter-temporal coordination incentives.

Linear Equilibrium: Optimal weights on private and public signals in $t=1$

In order to solve for the optimal actions in both periods, we start by deriving the optimal weights on private and public signals for uninformed agents and identify the optimal and the average action. By drawing upon equation (8), we write the optimal action for uninformed agents as

$$\begin{aligned} a_{i1}^{*} = r\left\{\rho\left[\frac{\gamma_1\beta}{\delta+\beta}x_i + \left(1-\frac{\gamma_1\beta}{\delta+\beta}\right)s_i\right] + (1-\rho)\left[\frac{\beta}{\delta+\beta}x_i + \left(1-\frac{\beta}{\delta+\beta}\right)s_i\right]\right\} \\ + (1-r)\left[\frac{\beta}{\delta+\beta}x_i + \left(1-\frac{\beta}{\delta+\beta}\right)s_i\right] \end{aligned} \tag{9}$$

The last term depicts the Bayesian considerations of the two pieces of information to form the expectation on θ. The first term contains the expectation about the average action in the preceding period and the expectation about the average action among the uninformed within the same period. How successful agents in $t=1$ can coordinate with agents from both periods depends on the correlation ρ. However, by the degree $(1-\rho)$ the signals are uncorrelated, i.e. s_i is predominantly private, and agents expect average actions to reflect the fundamental.

Derived in detail in appendix A, solving for the optimal weight on the private signal for uninformed agents yields

$$\gamma_1^* = \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}. \tag{10}$$

By symmetry, the optimal weight on the public signal is given by $(1-\gamma_1^*)$, respectively. In our framework, uninformed agents play the same game as the informed agents in CH, except that rather than receiving the public signal y, they receive the multiplier signal s_i, which becomes more public the smaller the idiosyncratic noise ($\sigma_\varphi^2 \to 0,\ \mu \to \infty$). The more public s_i becomes, the stronger its focal role for agents in $t=1$ and in case of perfect correlation initially uninformed agents become equally well informed. Therefore, the optimal weight on s_i increases in ρ and for $\rho \to 1$ approaches the optimal weight of the static MS case. On the contrary, for high idiosyncratic noise in s_i ($\sigma_\varphi^2 \to \infty,\ \mu \to 0$), agents fully rely on the private signal as $\gamma_1^* \to 1$.

Figure 1 illustrates the aforementioned results. It can be seen that the equilibrium weight on the private signal decreases in ρ and approaches 0 for $r=1$ and approaches 0.5 for $r=0$. For increasing ρ, the equilibrium weight on the private signal approaches the weight of the static MS case.

Conclusion 1: *The lower the idiosyncratic noise of the multiplier signal, the higher the correlation with the public signal and the stronger its focal role for uninformed agents.*

Now we can state the optimal actions for the subgame of uninformed agents as

$$a_{i1}^* = \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}x_i + \left[1-\frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\right]s \tag{11}$$

and the average action becomes

$$\begin{aligned}\bar{a}_1^* &= \int_P^1 a_{i1}^* \\ &= \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\theta_1 + \left[1-\frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\right]s \\ &= \frac{\delta s+\beta(1-r\rho)\theta}{\delta+\beta(1-r\rho)}. \end{aligned} \tag{12}$$

In appendix C, we show that that the linear equilibrium established here is the unique equilibrium of the subgame in $t=1$.

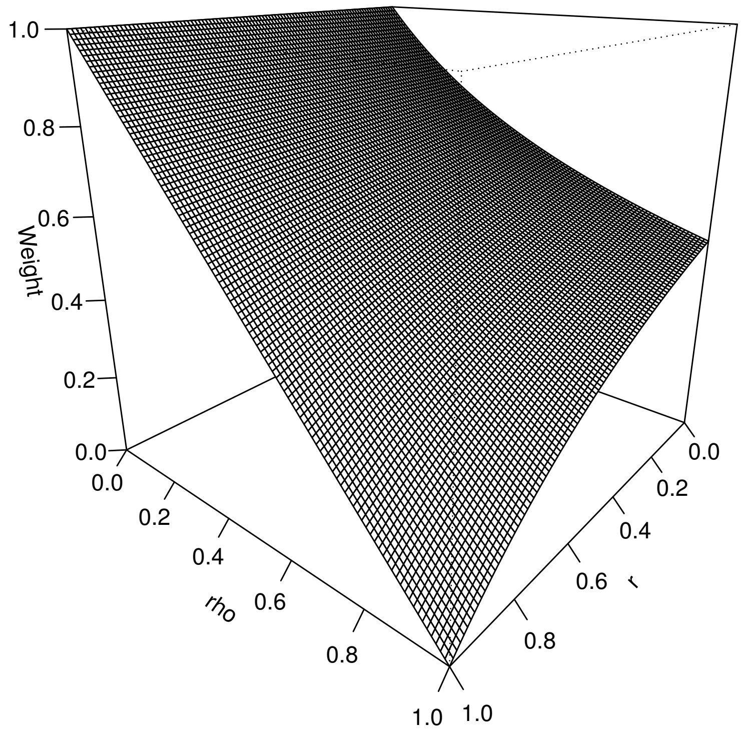


Figure 1: Optimal weight on private signal in $t = 1$

Note: the illustration assumes equal precision of the private and public signal $\beta = \alpha = 0.01$

Linear Equilibrium: Optimal weights on private and public signals in $t = 0$

The optimal action of informed agents based on (7) is given by

$$a_{i0}^* = (1-r)\left[\frac{\beta}{\alpha+\beta}x_i + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right] + rP\left[\frac{\gamma_0\beta}{\alpha+\beta}x_i + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right]$$
$$+ r(1-P)\left\{\rho\left[\frac{\gamma_0\beta}{\alpha+\beta}x_i + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right] + (1-\rho)\left[\frac{\beta}{\alpha+\beta}x_i + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right]\right\}. \quad (13)$$

The first term of this expression replicates the static MS case, whereby agents form Bayesian expectation about θ. The second term gives informed agents' incentive to account for average play in the same period, which is weighted by the coordination incentive parameterized by r and the corresponding fraction P of informed agents. The last term formulates the anticipation of the average action of the $(1-P)$ uninformed in the second period. If the correlation ρ between y and s_i is strong, the average actions in $t = 1$ and $t = 0$ become similar. Therefore agents form equal expectations for both periods (cp. expression after ρ). In contrast, for a weak correlation ρ, agents in $t=0$ expect the average action in $t=1$ to be close to the fundamental (cp. expression after $1-\rho$). In sum, the anticipation effect is accounted for by a correlation weighted average of the expectations on the fundamental and the average action in the first period. Solving for the optimal weight on the private signal for informed agents - detailed in appendix B - yields

$$\gamma_0^* = \frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}. \tag{14}$$

By symmetry, the optimal weight on the public signal can be written as $(1-\gamma_0^*)$. Optimal actions of the subgame in $t=0$ are then given by

$$a_{i0}^* = \frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}x_i + \left[1-\frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\right]y$$

and the average action becomes

$$\begin{aligned}
\bar{a}_0^* &= \int_P^1 a_{i0}^* \\
&= \frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\theta + \left[1-\frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\right]y \\
&= \frac{\alpha y+\beta[1-r(1-(1-\rho)(1-P))]\theta}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}.
\end{aligned} \tag{15}$$

In appendix D, we show that the linear equilibrium established here is the unique equilibrium of the subgame in $t=0$. Expressing $\bar{a}_0^*$ in terms of the random variables θ, η and ϵ_i yields

$$\bar{a}_0^* = \theta + \frac{\alpha\eta}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}.$$

In equilibrium, the average action of the informed agents is distorted away from the fundamental state according to the noise in the public signal η and the corresponding weight that is placed on the public signal y.

The optimal weights for informed agents show some interesting properties when compared to the static case in MS and CH. First, it can be seen that for $P=1$ - where all agents become informed and act in the first period - the optimal weight on the private signal collapses to the static MS case. Second, for any $0<P<1$, the optimal weight on the private signal depends on the correlation ρ between the public signal y and the multiplier signal s_i. For $\rho=1$, i.e. y and s_i are perfectly correlated and $\sigma_\varphi^2 \to 0$, the optimal weight replicates the weight in MS, since $[1-(1-\rho)(1-P)]=1$. Put simply, in the limit, informed agents anticipate that uninformed agents basically receive the same information, which is equivalent to the MS scenario where all agents get informed. Third, for $\rho=0$ – which is the case for $\sigma_\varphi^2 \to \infty$ – the equilibrium weight on the private signal replicates the weight in CH, since $[1-(1-\rho)(1-P)]=P$. Put simply, for the case that the multiplier signal is extremely imprecise, the sequential game replicates the static CH case as there are no multiplier effects and the multiplier signal adds nothing to the game.

Figure 2 illustrates the optimal weight on the private signal depending on the coordination motive r and the fraction P of agents being informed about the public signal y. It can be seen that for an increasing r, the optimal weight on the private signal decreases for all P. For $P = 1$, the optimal weight on the private signal reaches zero for maximum r. For $P < 1$, the optimal weight on the private signal remains significantly above zero. Even when coordination becomes the predominant motive in $t = 0$, informed agents do not assign full weight to the public signal due to a critical mass of uninformed agents relying on the multiplier signal, which imperfectly reflects the public signal y. For any intermediate value of ρ - i.e. for a reasonable correlation of

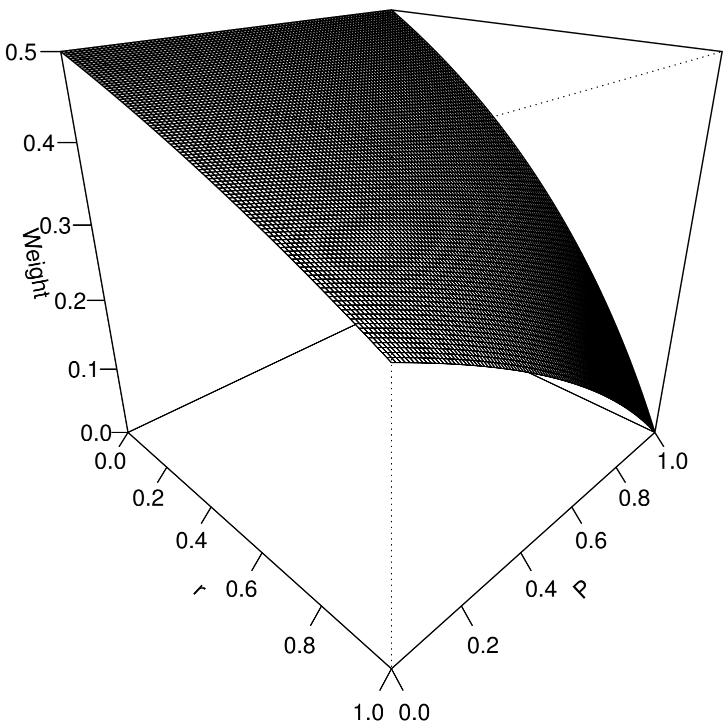


Figure 2: Optimal weight on private signal in $t = 0$

Note: the illustration assumes equal precision of the private and the public signal $\alpha = \beta = 0.01$ and a correlation of $\rho = 0.5$

the multiplier signal and the public signal - and for $0 < P < 1$, the optimal weight for informed agents on the private signal is higher than in the MS case, although - more importantly – it is lower than in the CH case. Figure 3 illustrates a straightforward comparison of the CH, the MS and our own case with respect to the optimal weight on the private signal of informed agents. This constitutes one of the central arguments of our paper, which can be summarized as follows: if informed agents expect uninformed agents to delay their action and learn about the public signal through the multiplier signal, the focal role of the public signal y is preserved by the multiplier signal s_i. Informed agents account for uninformed agents' getting informed and thus place a higher weight on the public signal and less on their private signal for any P. This holds true for multiplier signals that are not extremely imprecise.

Conclusion 2: *For reasonable correlation of the multiplier and the public signal, the optimal weight on the private signal is higher than in the static MS case but lower than in the CH case for any* $0 < P < 1$. *The public signal has a stronger focal role when introducing multiplier effects.*

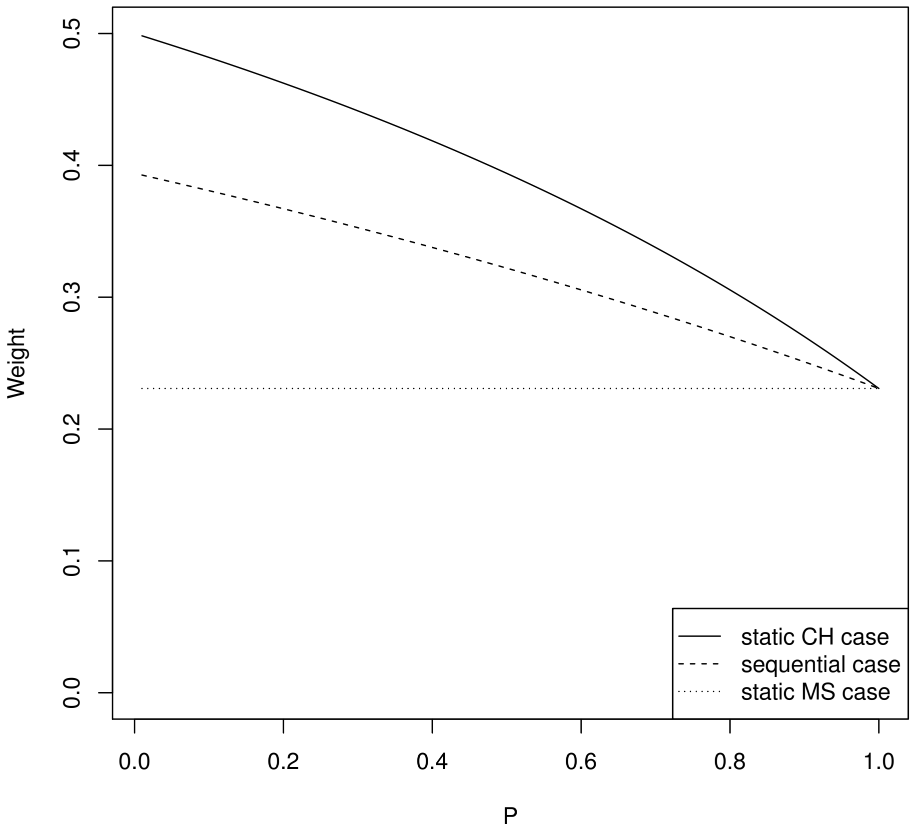


Figure 3: Optimal weight on private signal in comparison to CH and MS

Note: the illustration assumes equal precision of the private and the public signal $\alpha = \beta = 0.01$ and a correlation of $\rho = 0.5$ and $r = 0.7$

2 Welfare: The optimal degree of publicity

Our welfare specification follows the same formalization as in MS and CH, whereby coordination adds to utility on the individual level but not to aggregate welfare. Our model is thus prone to the criticism expressed by Woodford (2005): if coordination was a welfare objective per se, public information should always be released at maximum precision and publicity. Nonetheless, we follow the argument by CH: while agents individually benefit from accurately predicting expectations of other agents, efficiency requires actions to reflect the fundamental state and any form of overreaction to the public signal is socially undesirable.

Consider a pre-stage of our game where the central bank chooses a degree of publicity P to maximize expected welfare. As the central bank chooses P such that actions are close to the fundamentals, we can write

$$\begin{aligned} E[W(a,\theta)] &= -E[\int_{i=0}^{P}(a_{i0}-\theta)^2+\int_{P}^{1}(a_{i1}-\theta)^2] \\ &= -\int_{i=0}^{P}E\{[\gamma_0^* x_i+(1-\gamma_0^*)y-\theta]^2\}di-\int_{P}^{1}E\{[\gamma_1^* x_i+(1-\gamma_1^*)s_i-\theta]^2\}di \\ &= -P[\gamma_0^{*2}E(\epsilon_i^2)+(1-\gamma_0^*)^2E(\eta^2)]-(1-P)[\gamma_1^{*2}E(\epsilon_i^2)+(1-\gamma_1^*)^2E(\varphi^2+\eta^2)]. \end{aligned}$$

Now inserting optimal weights for both periods - γ_0^* from (14), γ_1^* from (10) - yields

$$\begin{aligned} = -P\left\{\left[\frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\right]^2\frac{1}{\beta}+\left[1-\frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\right]^2\frac{1}{\alpha}\right\} \\ -(1-P)\left\{\left[\frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\right]^2\frac{1}{\beta}+\left[1-\frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\right]^2\frac{1}{\delta}\right\}, \end{aligned}$$

which can be rearranged to obtain

$$= -P\frac{\beta[1-r(1-(1-\rho)(1-P))]^2+\alpha}{[\alpha+\beta(1-r(1-(1-\rho)(1-P)))]^2}-(1-P)\frac{\beta(1-r\rho)^2+\delta}{[\delta+\beta(1-r\rho)]^2}. \tag{16}$$

It can be seen that for $\sigma_\varphi^2\to\infty$, $\rho\to 0$ and $\delta\to 0$ the limit of the above expression becomes

$$= -P\frac{\beta(1-rP)^2+\alpha}{[\alpha+\beta(1-rP)]^2}-(1-P)\frac{1}{\beta}.$$

This expression replicates the expected welfare of the static CH case. Although agents act in different periods, the game collapses to the static case for a very imprecise multiplier signal compared to the private signal. The intuition of this result is straightforward: if the multiplier signal holds no value for the uninformed in $t=1$, they fully rely on their private signal, which in turn is anticipated by the first movers. We analyze expression (16) in further detail by carrying out various graphical illustrations with distinct parametrizations. We illustrate our findings graphically to enable an intuitive understanding since first derivatives of the expected welfare function are quite complex (see appendix E).

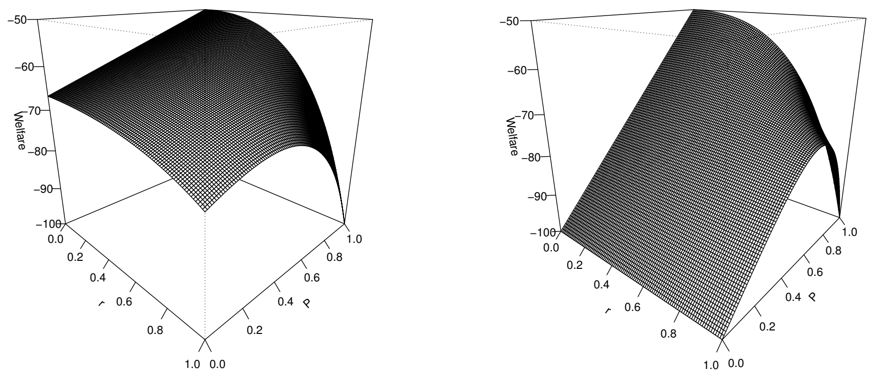
Comparing the static and dynamic case

Figure 4a depicts expected welfare with respect to P and r in our framework, while figure 4b depicts expected welfare as formulated in CH, which we identified as being equivalent to our case for $\delta \to 0$. For both illustrations, we assume equal precision of the private and public signal ($\alpha = \beta$) as well as a correlation between the public and multiplier signal of 0.5. In the static CH case, the optimal level of publicity is lower than 1 if, and only if, $\frac{\alpha}{\beta} < 3r - 1$, or for $\alpha = \beta$ if, and only if, $r > \frac{2}{3}$. In our sequential case, the optimal level of publicity not only depends on the coordination motive r and the relative precision of private and public signals; rather, optimal publicity crucially depends on the correlation of the multiplier and the public signal. For the case illustrated by figure 4a, the optimal publicity P is smaller than 1 and - more importantly - smaller than in the static CH case for any $r > 0$. The stronger the coordination motive, the higher the loss from a non-optimal level of publicity. Non-optimality thereby results from either too low or too high publicity .

For $r = 0$, $P = 1$ is welfare maximizing. Put simply, if there is no desire to coordinate, there is no overreaction to the public signal and the optimal information policy is to inform all agents. For $r \to 1$, $P = 0.38$ is welfare maximizing, which is lower than in the static CH case as the multiplier signal transmits disclosed public information from informed to initially uninformed agents. This transmission is anticipated and accounted for by the informed agents, the focal role of the central bank's signal grows stronger and – ceteris paribus – the optimal degree of publicity decreases.

Conclusion 3: *For equivalent precision of the private and public signal as well as a correlation between the multiplier and public signal of 0.5, the welfare maximizing degree of publicity is lower in the two-period case than in the static CH case.*

If the multiplier signal is only weakly correlated with the public signal (figure 5a), the optimal level of publicity P is higher than under strong correlation (figure 4a). Intuitively, the welfare maximizing central bank benefits from informing more agents due to the weaker focal role of the multiplier signal. As before, the stronger the coordination motive, the higher the loss from a non-optimal level of publicity. However, in this case, losses are asymmetrically distributed, i.e. publicity higher than the optimum is more welfare damaging compared to publicity lower than the optimum. If the multiplier signal is more strongly correlated with the public signal, the optimal level of publicity P substantially decreases. In our illustration of figure 5b, the multiplier signal is highly correlated and the focal role of the public signal continues to the second period and thus the optimal P decreases to 0.28.

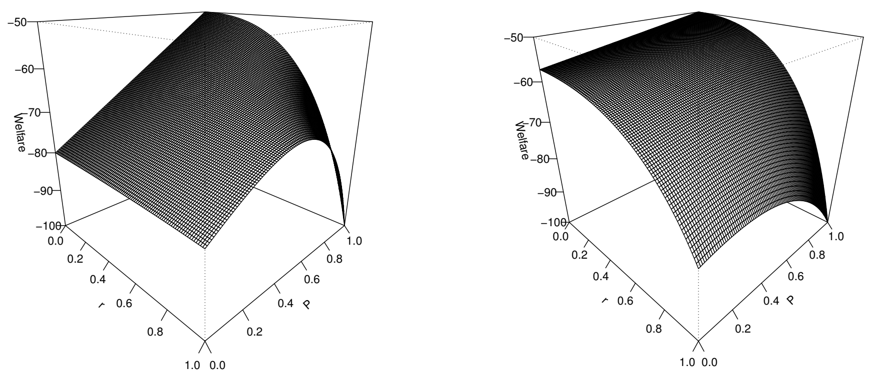


(a) sequential case (b) static CH case

Figure 4: Welfare depending on r and P for the sequential case

Note: the illustration assumes equal precision of the private and the public signal $\alpha = \beta = 0.01$ and a correlation of the multiplier s and the public signal y of $\rho = 0.5$

Conclusion 4: *For a weak correlation between the multiplier and the public signal, optimal publicity is high, whereas for a strong correlation the optimal publicity is low. Therefore, the effectiveness of the multiplier signal in transmitting the public signal determines optimal central bank policy with respect to publicity.*



(a) low correlation: $\rho = 0.25$ (b) high correlation: $\rho = 0.75$

Figure 5: Welfare depending on r and P for low and high correlation of s and y

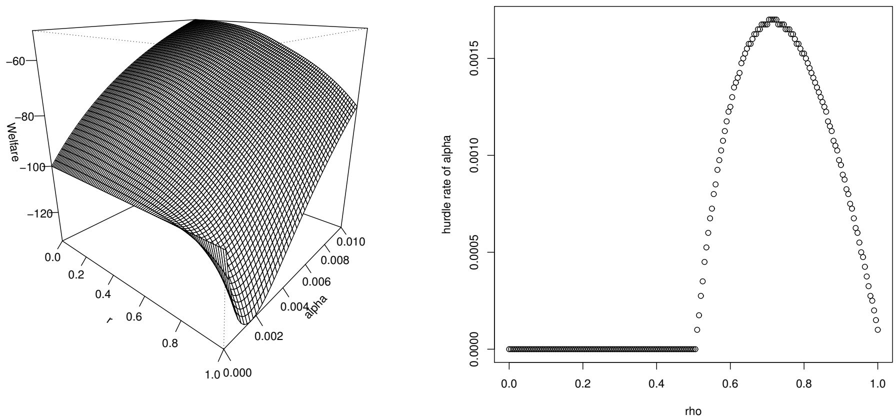
Note: the illustration assumes equal precision of the private and the public signal $\alpha = \beta = 0.01$

Optimal publicity and the precision of central bank's public signal

As CH note, if the public signal y can be provided with infinite precision $\alpha \to \infty$, the expected

welfare loss is zero and the first best solution is established. However, if there is an upper limit $\bar{\alpha}$ for the provision of the public signal, the central bank has to choose the optimal level of publicity and precision simultaneously. CH show that if the central bank can set $P = P^*$, the optimal precision is always maximal in the static case. The main theorem follows: public information should always be provided with maximum precision, but - under certain conditions - not to all agents. In our case, where the transmission of the public signal through the multiplier signal plays a crucial role for the optimal level of publicity, this theorem no longer holds. Figure 6a evaluates expected welfare at P^* with respect to α and r.
For any $\rho \leq 0.5$, overall welfare is strictly increasing in the precision of the public signal α. For any $\rho > 0.5$, the optimal precision only equals the maximal feasible one if the central bank can guarantee a minimum precision of the issued public signal. The public signal thus needs to reach a certain precision relative to the private signal. The hurdle rate of α gives the minimum precision above which higher public signal precision is strictly welfare increasing. Besides the coordination motive r, the hurdle rate depends on the correlation of the public and the multiplier signal (compare 6b). It can be seen that in our more general model, the main theorem of CH - public information should always be issued at maximum precision - only holds for $\rho < 0.5$. The intuition of this result evolves from the two types of publicity in our model. First, the publicity resulting from the fraction P of first-hand informed agents through y and second, the additional publicity resulting from the multiplier effect s_i informing initially uninformed agents in the second period. The later one is not in control of the central bank. Therefore, for high ρ, the multiplier effect counteracts the effect of reduced optimal publicity (P^*), which is set by the central bank for $r > \frac{2}{3}$ given $\alpha = \beta$. For a sufficiently strong multiplier effect and coordination motive, the central bank is incapable of reducing overall publicity to a level where the optimal precision of the public signal α remains the maximal feasible. In this case, the central bank has to reduce the precision to $\alpha = 0$, i.e. the central bank should release a signal which is of no informational value. These consideration connect to previous findings of "bang-bang solutions" for the precision of the public signal: precision should be chosen either maximal or at 0. In comparison to the "bang-bang solution" found in MS, in our model the finding only holds for $\rho > 0.5$. However, Svensson (2006) argues - based upon empirical findings on the precision of private versus public sector forecasts, e.g. by Romer and Romer (2000) - that it seems unlikely to assume the relative precision of public information being sufficiently weak for this finding to become relevant.

Conclusion 5: *Given optimal publicity P^*, the optimal precision of the public signal is at the maximum only if $\rho \leq 0.5$. For $\rho > 0.5$, it can be optimal to withhold very imprecise public information and issue a signal carrying no information at all.*



(a) public signal precision: α (b) hurdle rate of α depending on ρ

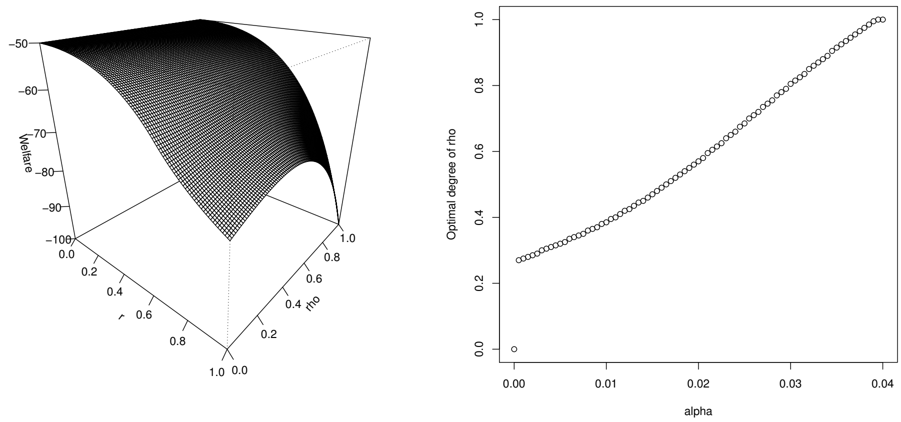
Figure 6: Welfare depending on r and α evaluated at $P = P^*$

Note: the illustration assumes a) $\beta = 0.01$ and $\rho = 0.75$ and b) $\beta = 0.01$ and $r = 0.99$

Desirability of the multiplier effect

On the one hand, the multiplier signal leads to the uninformed agents in our model being better informed than in the static CH case, whereby uninformed agents might more accurately match the fundamental state by their actions. On the other hand, the multiplier signal serves a focal role that fosters coordination and aligns the actions of all agents. Naturally, the question arises concerning whether the multiplier effect is generally desirable from a welfare perspective.
In our two-period beauty contest model, multiplier effects only add to overall welfare if the central bank chooses the optimal level of publicity ($P = P^*$) and the multiplier signal carries some information and is not extremely imprecise, i.e. $\rho > 0$. However, if the coordination incentive is sufficiently high ($r > 0.5$), there is a maximum degree of the multiplier effect that is smaller than 1 beyond which it harms overall welfare (compare Figure 7a). The intuition is the same as for the considerations of an optimal level of publicity of the public signal (P^*): A strong multiplier effect distorts actions substantially away from the fundamental and harms overall welfare. However, the optimal degree of the multiplier effect for high r depends on the precision of the public signal (compare Figure 7a). The more precise the public signal is, the stronger can the multiplier effect become without being detrimental to welfare. Precise public information should spread, while imprecise public information should not be subject to strong multiplier effects.

Conclusion 6: *Given a welfare optimizing level of publicity ($P = P^*$), the multiplier signal always adds to overall welfare only if the coordination incentive is sufficiently small. For strong coordination incentives, the maximum correlation of the multiplier signal up to which welfare increases, depends on the precision of the public signal.*



(a) multiplier effects: ρ (b) optimal degree of ρ

Figure 7: Welfare depending on r and ρ evaluated at $P = P^*$

Note: the illustration assumes a) $\alpha = \beta = 0.01$ and b) $\beta = 0.01$ and $r = 0.99$

This finding has an interesting implication for the information policy design of public authorities such as central banks. It supports the opinion that it might be optimal to provide central bank information gradually over time. For this to see, consider the public signal y and the multiplier signal s_i both being issued by the central bank. Thereby, the public signal can be interpreted as a public announcement such as a press conference following a monetary policy decision and, for instance, the multiplier signal represents subsequent interviews and speeches by central bank officials to further inform the public about the monetary policy stance. Given that the central bank chooses the optimal degree of publicity for informed agents in the first period ($P = P^*$), it can be optimal to spread the public information later on. Intuitively, uninformed agents being better informed through s_i by the central bank itself might overcompensate the potential distortions of a higher overall publicity of the public signal y. However, for a strong multiplier effect (high ρ), the reverse logic applies and distortions might overcompensate any positive effects of informing the uninformed. In other words, given high coordination incentives (high r), creating two smaller focal points lead to less distortions than informing all agents in the first place. This holds true as long as the central bank is able to limit the publicity of the multiplier signal and thus control multiplier effects to some extent.

3 Discussion

In this section, we discuss further issues related to our main framework. Firstly, we consider heterogeneous private signal precision between informed and uninformed agents. Secondly, we relate our model to the literature emphasizing asymmetric good and bad news reporting by the media. Finally, we discuss our theoretical findings in the light of experimental evidence on generalized beauty contests and social learning.

Heterogeneous private signal precision

An interesting variation of our model results from relaxing the assumption of equal private signal precision across periods. Consider that private signal precision is very low for uninformed agents, e.g. because private investors are not spending as many resources to gather information compared to institutional investors. It might plausibly be assumed that agents with more precise private information are more capable of receiving and processing public information and thus act prior to the uninformed agents with imprecise private information. Uninformed agents are subsequently prompted to closely follow the multiplier signal, where feedback trading might be considered a related phenomenon. Conversely, informed agents account for this reaction by assigning a higher weight on public information than under the assumption of uninformed agents acting upon private information of equal precision. Central bank communication has to account for the aggravated overreaction to public information in such an environment. The contrary logic applies if the private information of uninformed agents is more precise.

Asymmetric reporting by the media

We consider the media as a potential multiplier, which relates our model to the empirical literature investigating the role of the media in spreading economic news from public announcements. As previous studies suggest, the media reports more comprehensively on "bad" news than "good" news (cp. Hamilton 2004 and Soroka 2006), which has a substantial influence on the expectation formation process. Accordingly, the correlation of the multiplier signal generated by the media and the public signal is higher for bad news announced by the central bank. The focal role of public announcements becomes more pronounced in case of bad news than in case of good news. Therefore, optimal publicity depends on the content of the news itself as the content determines the intensity of the multiplier effect. Our model enables an optimal communication policy design to account for such inter-temporal dynamics.

Experimental evidence

Dale and Morgan (2012) and Cornand and Heinemann (2014) transfer the static game introduced by MS to a laboratory setting. They present experimental evidence on the focal role of public information, finding that agents significantly overweight public information compared to private information. However, the overreaction to public information is weaker than predicted by theory. Empirical weights are best predicted by a level-2 reasoning within a cognitive hierarchy model, which precludes public information from being detrimental to welfare Cornand and Heinemann (2013). Overall, the detrimental effect of public information is diminished when agents fail to form higher order beliefs. Baeriswyl and Cornand (2014) experimentally test the partial publicity argument introduced by CH. Comparing partial publicity - where only a fraction of all agents become informed - with partial transparency - where the public signal is disclosed to all agents, albeit with some ambiguity - they show that the two approaches are theoretically equivalent in their effectiveness of controlling overreaction. This equivalency resembles the results acquired in the lab. However, the authors argue that partial transparency is easier to implement and less discriminating as there are no uninformed agents. Experiments on social learning - a key multiplier effect - indicate that subjects are incapable of using Bayes rule optimally (Weizsäcker 2010). Agents cling to private information and underweight the value of social learning information. Social learning is insufficient, which results in informational inefficiencies. Transferring this core finding to our framework, we might similarly expect social learning to be weaker than predicted by theory. Agents might systematically underweight the information revealed by observable actions of the informed agents and cling to private information. These considerations connect to the finding of a weaker overreaction to public information in the lab than predicted by theory (Cornand and Heinemann 2014). In sum, the overreaction to publicly observable signals such as those issued by the central bank or derived by observable actions might be weaker than predicted by theory. Cognitive limitations or distrust in observable actions reduce the relevance of social learning yet increase the relevance of private information. However, since the interactions in our two-period framework are quite complex and anticipation effects might enter into play, future empirical research should explore the effectiveness of social learning and potential distortions.

4 Conclusion

We introduce a two-period generalized beauty contest to study the optimal level of publicity for central bank information that is prone to multiplier effects. We show that as long as the multiplier signal is not extremely imprecise, the optimal degree of publicity is lower in our sequential setting than in the static case. This finding is driven by the focal role of the publicly issued central bank signal for both the informed and uninformed agents.

Our results hold relevance for the optimal design of central bank communication. In choosing the degree of publicity, the central bank merely decides on the fraction of first-hand informed agents, while the uninformed agents will imperfectly learn about the public signal through multiplier effects. Learning by the uninformed agents through multiplier effects is anticipated and accounted for by the informed agents, prompting them to assign a higher weight to the central bank's signal. Accordingly, overreaction is substantially aggravated by multiplier effects as they lead to initially uninformed agents being involved in the coordination game. For a reasonably precise multiplier signal, the central bank has to reduce the publicity of its signal compared to the static case. Despite an optimal policy design, multiplier effects might decrease overall welfare for a strong coordination motive.

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Appendix A

Derivation of optimal weights on the private and the public signal in t=1

We start the derivation of optimal weights for uninformed agents by considering equation (9)

$$a_{i1}^* = r\left\{\rho\left[\frac{\gamma_1\beta}{\delta+\beta}x_i + \left(1-\frac{\gamma_1\beta}{\delta+\beta}\right)s_i\right] + (1-\rho)\left[\frac{\beta}{\delta+\beta}x_i + \left(1-\frac{\beta}{\delta+\beta}\right)s_i\right]\right\}$$
$$+(1-r)\left[\frac{\beta}{\delta+\beta}x_i + \left(1-\frac{\beta}{\delta+\beta}\right)s_i\right]$$

which can be rearranged such that

$$a_{i1}^* = \left(\frac{\beta + r\rho\gamma_1\beta - r\beta\rho}{\beta+\delta}\right)x_i + \left(1-\frac{\beta + r\rho\gamma_1\beta - r\beta\rho}{\beta+\delta}\right)s_i.$$

It can be seen that the two multipliers of x_i and s_i are perfectly symmetric. Since we assumed that all agents use their available information in a linear fashion (see equations (4) and (5)), we obtain

$$\gamma_1 = \frac{\beta + r\rho\gamma_1\beta - r\beta\rho}{\beta+\delta},$$

from which we can solve for the optimal weight on the private signal

$$\gamma_1^* = \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}.$$

By symmetry, the optimal weight on the multiplier signal s is then given by

$$(1-\gamma_1^*) = 1-\frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}.$$

Appendix B

Derivation of the optimal weight on the private and public signal in period t=0

We start the derivation of optimal weights for informed agents by considering equation (13)

$$a_{i0}^* = (1-r)\left[\frac{\beta}{\alpha+\beta}x_i + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right] + rP\left[\frac{\gamma_0\beta}{\alpha+\beta}x_i + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right]$$
$$+r(1-P)\left\{\rho\left[\frac{\gamma_0\beta}{\alpha+\beta}x_i + \left(1-\frac{\gamma_0\beta}{\alpha+\beta}\right)y\right] + (1-\rho)\left[\frac{\beta}{\alpha+\beta}x_i + \left(1-\frac{\beta}{\alpha+\beta}\right)y\right]\right\},$$

which can be rearranged such that

$$\begin{aligned} a_{i0}^* = & \left(\frac{\beta}{\alpha+\beta} + \frac{rP\gamma_0\beta}{\alpha+\beta} + \frac{r\rho\gamma_0\beta}{\alpha+\beta} - \frac{r\rho\beta}{\alpha+\beta} - \frac{r\rho P\gamma_0\beta}{\alpha+\beta} - \frac{rP\beta}{\alpha+\beta} + \frac{rP\rho\beta}{\alpha+\beta}\right) x_i \\ & + \left(1 - \frac{\beta}{\alpha+\beta} + \frac{rP\gamma_0\beta}{\alpha+\beta} + \frac{r\rho\gamma_0\beta}{\alpha+\beta} - \frac{r\rho\beta}{\alpha+\beta} - \frac{r\rho P\gamma_0\beta}{\alpha+\beta} - \frac{rP\beta}{\alpha+\beta} + \frac{rP\rho\beta}{\alpha+\beta}\right) y \end{aligned}$$

As in the case for uninformed agents, there is perfect symmetry in the multipliers of x_i and y. Since it is assumed that information is used in a linear fashion (see equations (4) and (5)), we obtain

$$\gamma_0 = \frac{\beta}{\alpha+\beta} + \frac{rP\gamma_0\beta}{\alpha+\beta} + \frac{r\rho\gamma_0\beta}{\alpha+\beta} - \frac{r\rho\beta}{\alpha+\beta} - \frac{r\rho P\gamma_0\beta}{\alpha+\beta} - \frac{rP\beta}{\alpha+\beta} + \frac{rP\rho\beta}{\alpha+\beta},$$

from which we can solve for the optimal weight on the private signal

$$\gamma_0^* = \frac{\beta[1 - r(1 - (1-\rho)(1-P))]}{\alpha + \beta[1 - r(1-\rho)(1-P))]}.$$

By symmetry, the optimal weight on the public signal y is then given by

$$(1 - \gamma_0^*) = 1 - \frac{\beta[1 - r(1 - (1-\rho)(1-P))]}{\alpha + \beta[1 - r(1-\rho)(1-P))]}.$$

Appendix C

Proof of Uniqueness for t=1

Here we prove that the equilibrium strategy identified for $t = 1$ is a linear combination of available signals x_i and s_i. The proof follows CH (p. 738). Intuitively, it has to be proven that the infinite sum of average expectations of the average expectations...(k times) is bounded and converges to the linear equilibrium identified in section 1.2. The optimal action of uninformed agents is given by the first-order condition of the utility function

$$a_{i1} = (1-r)E_i(\theta) + rE_i(\overline{a}).$$

The average action of all agents is given by

$$\begin{aligned} \overline{a} &= P\int_0^1 a_{i0}(x_i, y)di + (1-P)\int_0^1 a_{i1}(x_i, s_i)di \\ &= P\overline{a}_0(\theta, y) + (1-P)\overline{a}_1(\theta, s_i), \end{aligned}$$

which can be plugged into the FOC such that

$$a_{i1} = (1-r)E_{i1}(\theta) + rE_{i1}[P\overline{a}_0(\theta, y) + (1-P)\overline{a}_1(\theta_, s_i)]$$

Since $E_{i1}(\overline{a}_1) = E_{i1}(\overline{a}_0) = \rho\overline{a}_0(\theta, s_i) + (1-\rho)\theta$ we can write

$$\begin{aligned}
&= (1-r\rho)E_{i1}(\theta) + r\rho E_{i1}[\overline{a}_1(\theta, s_i)] \\
&= (1-r\rho)E_{i1}(\theta) + r\rho(1-r\rho)E_{i1}[\overline{E}_1(\theta)] + (r\rho)^2 E_{i1}[\overline{E}_1(\overline{a}_1|\theta, s_i)] \\
&= (1-r\rho)\sum_{k=0}^{\infty}(r\rho)^k E_{i1}[\overline{E}_1^k(\theta)] + \lim_{k\to\infty}(r\rho)^k E_{i1}[\overline{E}_1^k(\overline{a}_1|\theta, s_i)],
\end{aligned}$$

where $\overline{E}_1^k(\theta)$ denotes the average expectation of the average expectation... (k times) of θ and $\overline{E}_1^k(\overline{a}_1)$ is the average expectation of the average expectation... (k times) of the average action in $t = 1$. Since we assume $0 < r < 1, r\rho < 1$ and thus the limes term in the above expression is zero. The equation reduces to

$$= (1-r\rho)\sum_{k=0}^{\infty}(r\rho)^k E_{i1}[\overline{E}_1^k(\theta)].$$

Following Lemma 1 from MS (p. 1527), we can substitute $E_{i1}[\overline{E}_1^k(\theta)] = (1-\mu^{k+1})s_i + \mu^{k+1}x_i$ with $\mu = \frac{\beta}{\delta+\beta}$, which yields

$$\begin{aligned}
&= (1-r\rho)\sum_{k=0}^{\infty}(r\rho)^k(1-\mu^{k+1})s_i + \mu^{k+1}x_i \\
&= (1-r\rho)\left[s_i\sum_{k=0}^{\infty}(r\rho)^k - s_i\mu\sum_{k=0}^{\infty}(r\rho\mu)^k + x_i\mu\sum_{k=0}^{\infty}(r\rho\mu)^k\right].
\end{aligned}$$

Now, since $r\rho\mu < 1$, the infinite sums in the expression above $\sum_{k=0}^{\infty}(r\rho\mu)^k$ converge to $\frac{1}{1-r\rho\mu}$. Similarly, the expressen $\sum_{k=0}^{\infty}(r\rho)^k$ converges to $\frac{1}{1-r\rho}$. We can thus write

$$\begin{aligned}
&= (1-r\rho)\left[\frac{s_i}{1-r\rho} - \frac{s_i\mu}{1-r\rho\mu} + \frac{x_i\mu}{1-r\rho\mu}\right] \\
&= s_i - \frac{s\mu(1-r\rho)}{1-r\rho\mu} + \frac{x_i\mu(1-r\rho)}{1-r\rho\mu}.
\end{aligned}$$

Inserting $\mu = \frac{\beta}{\delta+\beta}$ and rearranging terms yields

$$= \left[1 - \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}\right]s + \frac{\beta(1-r\rho)}{\delta+\beta(1-r\rho)}x_i,$$

which replicates the linear equilibrium established in section 1.2 and completes the proof.

Appendix D

Proof of uniqueness for $t = 0$

The proof of uniqueness of the equilibrium strategy in $t = 0$ follows the same logic as for $t = 1$. The optimal action of informed agents is given by the first-order condition of the utility function

$$a_{i0} = (1-r)E_i(\theta) + rE_i(\overline{a}).$$

The average action is given by

$$\begin{aligned}\overline{a} &= P\int_0^1 a_{i0}(x_i, y)di + (1-P)\int_0^1 a_{i1}(x_i, s_i)di \\ &= P\overline{a}_0(\theta, y) + (1-P)\overline{a}_1(\theta, s_i),\end{aligned}$$

which can be plugged into the FOC such that

$$a_{i0} = (1-r)E_{i0}(\theta) + rE_{i0}[P\overline{a}_0(\theta, y) + (1-P)\overline{a}_1(\theta, s_i)].$$

Since $E_{i0}(\overline{a}_1) = \rho\overline{a}_0(\theta, y) + (1-\rho)\theta$, we can write

$$= (1-r)E_{i0}(\theta) + rPE_{i0}[\overline{a}_0(\theta, y)] + r(1-P)(1-\rho)E_{i0}(\theta) + r(1-P)\rho E_{i0}[\overline{a}_0(\theta, y)].$$

Summarizing terms yields

$$\begin{aligned}&= [1 - r(1-(1-\rho)(1-P))]\, E_{i0}(\theta) + r[1-(1-\rho)(1-P)]E_{i0}[\overline{a}_0(\theta, y)] \\ &= [1 - r(1-(1-\rho)(1-P))]\, E_{i0}(\theta) + r[1-(1-\rho)(1-P)]\, [1 - r(1-(1-\rho)(1-P))]\, E_{i0}[\overline{E}_0(\theta)] \\ &\quad + [(r(1-(1-\rho)(1-P))]^2\, E_{i0}[\overline{E}_0(\overline{a}_0|\theta, y)] \\ &= [1 - r(1-(1-\rho)(1-P))] \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))]^k\, E_{i0}[\overline{E}_0^k(\theta)] \\ &\quad + \lim_{k\to\infty} [r(1-(1-\rho)(1-P))]^k\, E_{i0}[\overline{E}_0^k(\overline{a}_0|\theta, y)],\end{aligned}$$

where $\overline{E}_0^k(\theta)$ denotes the average expectation of the average expectation... (k times) of θ_0 and $\overline{E}_0^k(\overline{a}_0)$ is the average expectation of the average expectation... (k times) of the average action in $t = 0$. Since we assume $0 < r < 1, r[1-(1-\rho)(1-P)] < 1$ and thus the limes term in the above expression is zero. The equation reduces to

$$= [1 - r(1-(1-\rho)(1-P))] \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))]^k\, E_{i0}[\overline{E}_0^k(\theta)].$$

Following Lemma 1 from MS (p.1527), we can substitute $E_{i0}[\overline{E}_0^k] = (1-\mu^{k+1})y + \mu^{k+1}x_i$ with $\mu = \frac{\beta}{\alpha+\beta}$ which yields

$$
\begin{aligned}
&= [1 - r(1-(1-\rho)(1-P))] \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))]^k \, (1-\mu^{k+1})y + \mu^{k+1}x_i \\
&= [1 - r(1-(1-\rho)(1-P))] \left\{ y \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))]^k - y\mu \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))\mu]^k \right. \\
&\quad \left. + x_i \mu \sum_{k=0}^{\infty} [r(1-(1-\rho)(1-P))\mu]^k \right\}.
\end{aligned}
$$

Now, since $r[1-(1-\rho)(1-P)]\mu < 1$, the infinite sums in the last expression above $\sum_{k=0}^{\infty}[r(1-(1-\rho)(1-P))\mu]^k$ converge to $\frac{1}{1-r[1-(1-\rho)(1-P)]\mu}$. Similarly, the expression $\sum_{k=0}^{\infty}[r(1-(1-\rho)(1-P))]^k$ converges to $\frac{1}{1-r[1-(1-\rho)(1-P)]}$ and we obtain

$$
\begin{aligned}
&= [1 - r(1-(1-\rho)(1-P))] \\
&\quad \left\{ \frac{y}{1-r[1-(1-\rho)(1-P)]} - \frac{y\mu}{1-r[1-(1-\rho)(1-P)]\mu} + \frac{x_i\mu}{1-r[1-(1-\rho)(1-P)]\mu} \right\} \\
&= y - \frac{y\mu[1-r(1-(1-\rho)(1-P))]}{1-r[1-(1-\rho)(1-P)]\mu} + \frac{x_i\mu[1-r(1-(1-\rho)(1-P))]}{1-r[1-(1-\rho)(1-P)]\mu}
\end{aligned}
$$

Inserting $\mu = \frac{\beta}{\alpha+\beta}$ and rearranging terms yields

$$
= \left[1 - \frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]}\right] y + \frac{\beta[1-r(1-(1-\rho)(1-P))]}{\alpha+\beta[1-r(1-(1-\rho)(1-P))]} x_i,
$$

which replicates the linear equilibrium established in section 1.2 and completes the proof.

Appendix E

Here, we present the first derivative of the welfare function

$$E[W(a,\theta)] = -P\frac{\beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]^2+\alpha}{\left\{\beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]+\alpha\right\}^2} - \left(1-P\right)\frac{\beta\left(1-r\rho\right)^2+\delta}{\left[\beta\left(1-r\rho\right)+\delta\right]^2} \tag{17}$$

with respect to $0 < P \leq 1$. We obtain

$$\begin{aligned}\frac{\delta E(W)}{\delta P} = \; & \frac{2\beta\left(1-\rho\right)rP\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right] - \beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]^2-\alpha}{\left\{\beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]+\alpha\right\}^2} - \frac{2\beta\left(1-\rho\right)rP\left\{\beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]^2+\alpha\right\}}{\left\{\beta\left[1-r\left(1-\left(1-\rho\right)\left(1-P\right)\right)\right]+\alpha\right\}^3} \\ & +\frac{\beta\left(1-r\rho\right)^2+\delta}{\left[\beta\left(1-r\rho\right)+\delta\right]^2}.\end{aligned}$$

The first two terms of the derivative correspond to the first part of the welfare function (loss in $t = 0$), while the last term corresponds to the second part (loss in $t = 1$).