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Renewable Resource Use with Imperfect Self-Control

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Abstract. We investigate renewable resources use when the harvesting agents face self-control problems. Individuals are conceptualized as dual selves. The rational long-run self plans for the infinite future while the affective short-run self desires to maximize instantaneous profits or utility. Depending on the degree of self-control, actual behavior is partly driven by short-run desires. This modeling represents impatience and present bias without causing time inconsistent decision making. In a model of a single harvesting agent (e.g. a fishery), we discuss how self-control problems affect harvesting behavior, resource conservation, and sustainability and discuss policies to curb overuse and potential collapse of the resource due to present-biased harvesting behavior. We then extend the model to several harvesting agents and show how limited self-control exacerbates the common pool problem. Finally, we investigate heterogenous agents and show that there are spillover effects of limited self-control in the sense that perfectly rational agents also behave less conservatively when they interact with agents with imperfect self-control.

Keywords: self-control, temptation, renewable resource use, sustainability, common pool resource management.

JEL: D60, D90, Q20, Q50, Q58, O40.

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1. Introduction

The way individuals evaluate present and future payoffs affects all dynamic economic decisions but it is perhaps of highest importance in the management of natural resources because here it may cause irrecoverable destruction of habitats and harvesting opportunities. So far, most theories of renewable resource use either assumed that individuals are purely myopic or discount the future exponentially, i.e. at a constant rate (e.g. Clark, 1973; Munro and Scott, 1985; Conrad and Clark, 1988). Exponential discounting is analytically convenient but it is potentially problematic from an empirical viewpoint. Research in psychology and behavioral economics suggests that individuals are frequently better represented by present-biased preferences that put particularly high weight on immediate gratification although they do not completely disregard future payoffs (see Frederick et al., 2002, and DellaVigna, 2009, for surveys).

One possibility to represent present-bias is by implementing (quasi) hyperbolic discounting (Ainslie, 1975). In economic life-cycle models it has been shown that individuals save and invest too little when they discount the future hyperbolically (e.g. Laibson, 1997, 1998). Conventional hyperbolic discounting, however, involves another problem, time-inconsistent decision making. It is thus a priori unclear whether inferior investment is caused by the declining discount rate as such or by the involved time inconsistency and reversal of plans. While it is widely believed that hyperbolic discounting necessarily involves time-inconsistency (e.g. Angeletos et al., 2001), it is actually possible to propose empirically plausible forms of hyperbolic discounting that support time-consistent decisions by giving up the stationarity assumption (Halevy, 2015). Such preferences are characterized by a discount factor that is multiplicatively separable in planning time and payoff time (Burness, 1976; Drouhin, 2020). They imply that individuals become more patient as they grow older, a feature which receives empirical support (Green et al., 1994; Bishai, 2004) and which is consistent with theoretical considerations on the evolution of time preference through natural selection (Rogers, 1994).

In the context of renewable resource use, time-consistent hyperbolic discounting implies that individuals use resources more conservatively. In fact, hyperbolic discounting abolishes the threat of extinction and leads in the long-run to the social optimum, i.e. it establishes the Green Golden Rule (Strulik, 2020). The intuition for this non-obvious result can be understood after inspection of the transitional dynamics of resource stock and discount rate. The first order conditions of any conventional problem of optimal control exclude extinction of renewable resources in finite time because it would imply a jump in the control variable and violate the optimality conditions. Thus, only asymptotic extinction could be an optimal outcome. When the discount rate declines hyperbolically, however, there exists always a finite point in time at which the discount rate falls short of any positive growth rate of the resource, a condition that eliminates asymptotic extinction and ensures sustainability. With further declining discount rate, the harvesting behavior converges towards the long-run social optimum (Strulik, 2020).

Duncan et al. (2011) show that time-inconsistent hyperbolic discounting can lead to over-exploitation and asymptotic extinction of renewable resources. These undesirable outcomes, however, need to be attributed to time inconsistency and the continuous reversal of resource management plans and not to the feature of hyperbolically declining discount rates. Together with the results from Strulik (2020), this suggests that hyperbolic discount rates are perhaps not the best way to study cases of resource mismanagement that are *caused* by present-biased preferences.

Motivated by these observations, we here propose a different approach to study the impact of present bias on resource use, based on the dual-self model of Thaler and Shefrin (1981) and Fudenberg and Levine (2006). These studies take into account insights from psychology and neurology showing that different areas of the brain are occupied with short-run (impulsive) behavior and long-run (planned) behavior. Humans are conceptualized as neither being mere "cold" long-run planners nor mere "hot" affective persons. The dual self consists of a rational long-run self who imperfectly controls the impulsive actions of a short-run self. Self-control incurs a utility cost, which is individual specific and generally increasing in the deviation of the constrained optimal solution from the solution preferred by the short-run self. Structurally, the dual-self model is isomorph to the temptation utility model of Gul and Pesendorfer (2001, 2004). The conventional solution of the discounted utility model is included as a special case of perfect self-control. Here we discuss for the first time the dual-self theory in an environmental economics context.

As an illustrative example, imagine a fisherman who correctly computes the daily catch that is consistent with the long-run optimal use of the fishery. Suppose that, after a few hours of fishing, the fisherman has already reached the long-run optimal catch and the long-run self suggests to call it a day and land the catch. The short-run self, excited by the fishing success, demands to carry on until the short-run profit maximum is reached. The degree to which the (long-run self of the) fisherman gives in to short desires and continues fishing depends on the personal level of self-control, a preference parameter given at this stage of economic analysis (but perhaps malleable in childhood; Mishel, 2014). The fisherman faces this problem every day anew and the decision involves no time inconsistency. It constitutes a particularly mild form of bounded rationality with potentially severe implications on renewable resource use.

A series of empirical studies have provided evidence for imperfect self-control as a driving force of impulsive consumption and low investment in general (Shiv and Fedorikhin, 1999; Baumeister, 2002; Ameriks et al., 2007). To our best knowledge, no study has so far investigated empirically the role of self-control in natural resource management. There exists some evidence that time preference and present bias is particularly high among fishermen (Johnson and Saunders, 2014) and that fishermen who displayed high discount rates in lab experiments extract common pool resources more excessively (Fehr and Leibbrandt, 2011). Huang and Smith (2014) provide evidence that fishermen respond to the presence of other common pool users by exerting more harvesting effort, as predicted by feedback strategies in dynamic common pool games. Hernuryadin et al. (2019) compare discount rates in fisheries and agrarian societies in Indonesia and document substantially higher discount rates among fishermen. This finding is particularly interesting in light of the study of Galor and Ozak (2016) who argue that historical exposure to higher crop yields and the associated experience of high returns of agricultural investments was conducive to the evolution of long-term orientation in agrarian societies. Applying the argument to marine societies, it could be argued that fishermen are less exposed to a waiting period for harvest. Harvesting may actually happen year round, the daily catch is sold completely on the market, and no part of the harvest needs to be saved as investment for the next harvesting season. It could thus be argued that, historically, fishermen experienced less reward for waiting and have thus evolved less patience and greater present-bias.

In the next Section we introduce the dual-self theory into the dynamic Schaefer-Gordon model and analyze harvesting behavior at the steady state as well off-steady-state dynamics. We show that greater self-control problems lead to larger harvesting shares and may cause asymptotic extinction of the renewable resource. We then compute the ad-valorem tax on the harvested product (the fish landing tax) that corrects present bias and establishes the long-run optimal solution. The optimal tax rate increases in the severity of self-control problems and declines in the level of the resource. The optimal tax rate ceases to exists if the resource declines below a lower limit. Taxation, however, continues to be an effective mean to curb overuse of the resource and can lead to a gradual recovery of the resource to a level at which the optimal tax becomes implementable again. We then extend the analysis towards several non-cooperative agents harvesting a common pool resource. In order to obtain an analytic solution for feedback (Markovian) strategies in the dynamic game, we linearize natural resource growth. We show that limited self-control amplifies the incentive for resource extraction and exacerbates the tragedy of the commons. Again we compute taxes that correct overexploitation due to present-bias and also optimal taxes that correct additionally overexploitation due to missing property rights. Finally, we investigate the interaction of two harvesting agents with different degrees of self-control problems. Specifically, we show that the self-control problem of one agent partly spills over to another agent of perfect self-control who is induced to harvest more than he would if he shared the common pool with an agent of perfect self-control. Since spillovers are incomplete, lower self-control of one agent leads to less harvest for the perfectly rational agent. In a heterogenous group, perfectly rational agents thus bear the costs of excessive harvesting behavior of boundedly rational agents.

2. The Basic Model

2.1. **Setup of the Model.** Consider growth of a renewable resource according to the Verhulst (1938) model (see e.g. Wilen, 1985). Absent of any harvesting, the resource stock, denoted by x, grows logistically until it reaches its carrying capacity κ. If undisturbed by harvesting, the change of the resource stock is given by $g(x) = rx(1 - x/\kappa)$ where r denotes the maximum natural growth rate. The maximum sustainable yield from the resource is attained where $g(x)$ reaches a maximum, i.e. where $g'(x) = 0$. Let the harvest level be denoted by h such that the change of stock is obtained as

$$\dot{x} = rx\left(1 - \frac{x}{\kappa}\right) - h. \quad (1)$$

Consider a firm that has the property rights to harvest a local resource pool and takes prices as given on a market with many other firms which each have monopoly access to their specific local resource pool. This approach is known as the (dynamic) Schaefer-Gordon model (Munro and Scott, 1985). In the Schaefer-Gordon model these firms are usually considered as fisheries. The firm maximizes profits $\pi = [p(1-\tau) - c(x, h)]h$, in which h is the size of the harvest, p is the price, τ is an ad-valorem tax rate and $c(x, h)$ are the unit costs of harvesting. Harvesting costs are declining in the stock because fish are easier to catch when they are plentiful. In deviation from the canonical Schaefer-Gordon model, we also assume that unit costs are increasing in the firm's harvesting effort h. This plausible assumption avoids bang-bang solutions, generates a unique interior solution as long as prices are not too large, and supports smooth transitional dynamics. For simplicity, we assume that $c(x,h) = \alpha h/x, \ \alpha > 0$.

The short-run temptation faced by harvesting agents is conceptualized as the short-run profits that they could generate by disregarding resource dynamics and sustainability issues. This interpretation is closest to the original formulation of temptation utility (Pesendorfer and Gul, 2001, 2004). Maximizing short run profits leads to the solution $h = p(1-\tau)x/(2\alpha)$. For sufficiently high costs, $\alpha > p(1-\tau)/2$, this solution is interior, $h < x$, such that even the short-run minded firm does not harvest the total stock in one instant of time. Profits of the interior solution are obtained as $\pi^s = p^2(1-\tau)^2 x/(4\alpha)$.

The harvesting agents are, however, not myopic but take long-run sustainability into account. Specifically, the long-run self of a harvesting agent considers the resource constraint and the discounted stream of all future profits. However, the long-run self experiences a utility cost from self-control that arises because he harvests less and makes less profits than desired by the short-run self. In order to translate profits in terms of utility we assume that agents have linear utility functions and that all income of harvesting agents (fishermen) stems from selling the harvest (the catch of the day). Applying Fudenberg and Levine's (2006) notion of self-control, this means that harvesting agents maximize

$$V = \int_0^\infty \mathrm{e}^{-\rho t} \left\{ \pi(x,h) - \omega \left[\pi^s(x) - \pi(x,h) \right] \right\} \mathrm{d}t \tag{2}$$

subject to (1). The term in square brackets reflects the difference between the utility desired by the short-run self and the actually experienced utility. The parameter ω measures the cost of self-control, $\omega \geq 0$. For $\omega = 0$, we have the special case of perfect self-control. The expression $\omega \left[\pi(x,h) - \pi^s \right]$ measures the total cost of restraining short run desires and not realizing maximum profits (e.g. from stopping fishing after two hours per day). Notice that maximizing V is equivalent to maximizing $\tilde{V} = \int_0^\infty \mathrm{e}^{-\rho t} \left\{ \pi(x,h) - \Omega \pi^s(x) \right\} \mathrm{d}t$, in which $\Omega = \omega/(1+\omega)$ measures individual susceptibility to temptation and $\Omega \pi^s(x)$ is the strength of temptation. This re-interpretation according to Gul and Pesendorfer (2001, 2004) maps the self-control parameter $\omega \in [0, \infty)$ onto a temptation parameter $\Omega \in [0,1)$, implying $\Omega \to 1$ for minimum self-control.

Inserting the definition of profits and the profits desired by the short-run self, the Hamiltonian for this problem is

$$\mathcal{H} = (1+\omega)\left[p(1-\tau)h - \frac{\alpha h^2}{x}\right] - \omega\left[\frac{p^2(1-\tau)^2}{4\alpha}\right]x + \lambda\left[rx\left(1-\frac{x}{\kappa}\right) - h\right].$$

Notice that the desire to harvest more than the long-run optimal level is increasing in the price of the resource and in the available stock of the resource. The first order condition for h and the associated costate equation are

$$(1+\omega)\left[p(1-\tau) - \frac{2\alpha h}{x}\right] - \lambda = 0, \tag{3}$$

$$(1+\omega)\frac{\alpha h^2}{x^2} - \omega\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + \lambda r\left(1-2\frac{x}{\kappa}\right) = -\dot{\lambda} + \lambda\rho. \tag{4}$$

It turns out that the solution is easier interpretable when we consider dynamics in the x-b–space, where $b = h/x$ is the share of the harvest. From (3) follows $\dot{\lambda} = -(1+\omega)2\alpha\dot{b}$. Using this information and substituting λ, equation (4) can be written as

$$2\alpha\dot{b} = \alpha b^2 - \frac{\omega}{1+\omega}\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + [p(1-\tau) - 2\alpha b]\left[r\left(1-2\frac{x}{\kappa}\right) - \rho\right] \tag{5}$$

System dynamics are fully described by (1) with $h = bx$ and (5), the initial stock $x(0)$, the boundary conditions $x \geq 0$, $b \geq 0$, and the transversality condition $\lim_{t\to\infty}\lambda(t)x(t)\mathrm{e}^{-\rho t} = 0$.

2.2. **Steady State and Sustainability.** To compute the steady state values (x^*, b^*), we solve (1) and (5) with $\dot{x} = \dot{b} = 0$ and obtain

$$b^*_{1,2} = \frac{2(\alpha(r+\rho) + p(1-\tau)) \pm \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + (4-3\Omega)p^2(1-\tau)^2}}{6\alpha} \tag{6}$$

$$x^*_{1,2} = \frac{r - b^*_{1,2}}{r}\kappa. \tag{7}$$

The following proposition, proven in the Appendix, shows that we can confine the analysis to the second steady state $(x^*_2, b^*_2) = (x^*, b^*)$.

Proposition 1. *The steady state (x^*_1, b^*_1) is a local minimum, whereas the steady state (x^*_2, b^*_2) is a local maximum of the Hamiltonian $\mathcal{H}$.*

Inspection of (7) shows that sustainability, defined as a positive steady state for the resource stock, requires $r > b^*$, i.e. that the natural growth rate of the resource exceeds the harvesting rate. Therefore, sustainability requires that the natural growth rate of the resource is sufficiently large, namely

$$r > \rho + \frac{p(1-\tau)}{2\alpha} - \frac{1}{2}\sqrt{4\rho^2 + \frac{p^2(1-\tau)^2(1-\Omega)}{\alpha^2}}. \tag{8}$$

The fact that the root in the negative last term of (8) declines in Ω shows that there are situations (parameter constellations) in which sustainability is realized with perfect self-control but not with imperfect self-control of fishermen. Greater self-control problems reduce the parameter set that supports sustainability. For minimal self-control, $\Omega = 1$, and (8) reduces to $r > p(1-\tau)/(2\alpha)$, requiring that the natural growth rate of the resource exceeds the harvest rate of a purely myopic fisherman. We also see that sustainability can always be realized by a sufficiently high tax rate τ. The next proposition, proven in the Appendix, considers the comparative statics of the steady state.

PROPOSITION 2. *The steady state harvest rate b^* increases in the severity of self control Ω, in the natural growth rate r, in the price p as well as in the time preference rate ρ. It decreases in the tax rate τ and in the extraction cost parameter α.*

We next characterize conditions for which the harvest rate is particularly affected by (the loss of) self-control, as shown in the following proposition (proved in the Appendix).

PROPOSITION 3. *The impact of self-control on the steady-state harvest rate increases in impatience ($\partial^2/\partial\Omega\partial\rho > 0$), the resource price ($\partial^2/\partial\Omega\partial p > 0$), and the natural growth rate of the resource ($\partial^2/\partial\Omega\partial r > 0$. It declines in the cost parameter ($\partial^2/\partial\Omega\partial\alpha < 0$).*

High prices and low costs increase instantaneous profits and therewith the temptation to harvest exceedingly much. Interestingly, a high natural growth rate of the resource also increases the impact of self-control because it reduces the opportunity cost of exerting self-control due to fast regrowth of the overexploited resource. Low self-control and impatience (measured by a high time preference rate) re-enforce each other in their impact on the harvest rate.

In the following we consider adjustment dynamics of the economy. The phase diagram in Figure 1 is constructed for a situation where the sustainability condition (8) is fulfilled for b^*. According to equation (1) there exist two $\dot{x} = 0$-isoclines. The first one is the ordinate, whereas the second one is given by $b = r(1 - x/\kappa)$. This is a linear function with negative slope $-r/\kappa$, which meets the ordinate at r and the abscissa at κ. Above the isocline, x decreases, whereas it increases below, as indicated by the arrows of motion. The $\dot{b} = 0$-isocline is a hyperbole with the asymptote $b = (1-\tau)p/2\alpha$. It intersects the abscissa at $\tilde{x} \equiv \kappa(r - \rho - \Omega p(1-\tau)/4\alpha)/(2r)$. If b is above the asymptote, then b increases if x is above the $\dot{b} = 0$-isocline and b decreases if x is below the $\dot{b} = 0$-isocline. If b is below the asymptote, dynamics of b are in the opposite direction, as indicated by the arrows of motion. The steady states of the economy are at the intersections of the $\dot{x} = 0$- and the $\dot{b} = 0$-isoclines. We observe two possible non-trivial steady states. The steady state with the smaller share of harvest is the local maximum, whereas the other one is the local minimum. Transitional dynamics, indicated by the arrows of motion, clearly show that the local maximum is a saddle point. System dynamics follow the saddlepath to the local maximum, as indicated in Figure 1. At the steady state the transversality condition is fulfilled since x and b and thus λ are constant. All other system dynamics either lead in finite time to the local minimum or to $b = 0$, a situation that leads to zero profits in finite time (and thus violates the transversality condition).

If individuals exhibit low self-control, sustainability may cease to exist. Such a situation is depicted in Figure 2. Here, the original situation with a positive long-run steady state is assigned to a low value of ω, i.e. a high degree of self-control. Declining self-control adds a term inversely proportional to $p(1-\tau)/\alpha - 2b$ to the $\dot{b} = 0$-isocline, which is diagrammatically represented by a leftward shift and widening of the $\dot{b} = 0$-isocline. It implies that the steady state harvesting ratio b^* increases and the resource stock x^* declines until eventually the positive saddlepoint ceases to exist. Such a scenario is shown by dashed lines in Figure 2. The changed system dynamics are indicated by dashed arrows of motion. The only situation compatible with the transversality rule is asymptotic convergence towards extinction.

2.3. **Optimal tax rate.** The tax rate τ can be set to nudge the self-control afflicted resource owner to behave in a long-run optimal way (assuming that the firm's long-run discount rate coincides with the social discount rate of a benevolent planner such that all over-exploitation can be attributed to present-bias induced from limited self-control). To find the optimal tax rate, we do not have to solve the problem explicitly for the harvesting decision. We can simply exploit the fact that the social planner's solution is contained in (5) for $\omega = \tau = 0$, since the long-run planner faces no self-control problem. Thus, the right hand side of (5) for $\tau = \omega = 0$ must equal the right hand side of (5) for $\omega > 0, \tau > 0$. This leads to

$$A(1-\tau)^2 + \tau B = 0 \quad \text{with} \quad A \equiv \frac{\omega}{1+\omega}\frac{p^2}{4\alpha} \quad \text{and} \quad B \equiv p\left[r\left(1 - 2\frac{x}{\kappa}\right) - \rho\right], \tag{9}$$

Figure 1. Phase Diagram

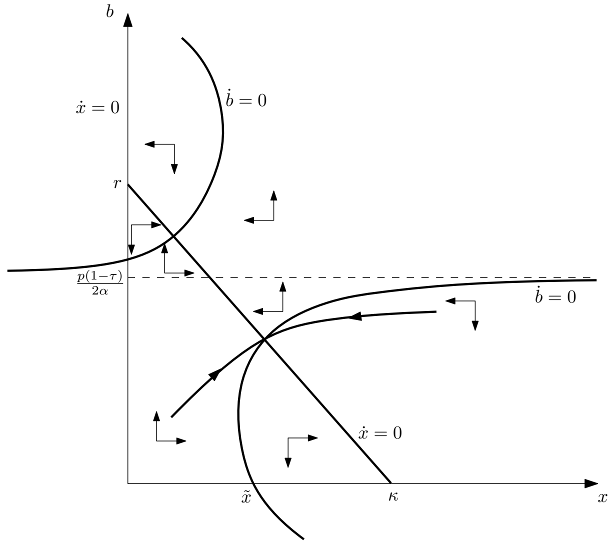
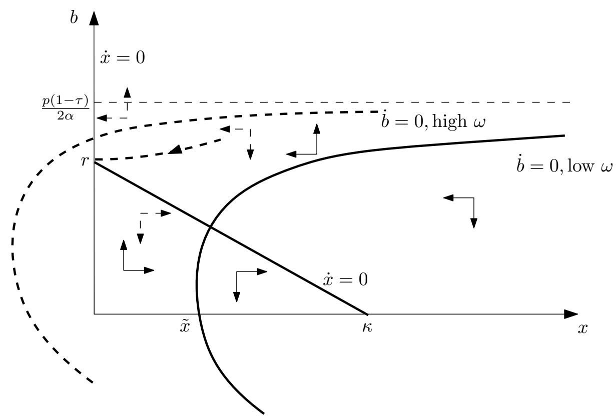


Figure 2. Phase Diagram: Declining Self-Control and Extinction



which gives the optimal tax rate

$$\tau^* = \frac{2A - B - \sqrt{B(B - 4A)}}{2A}.$$

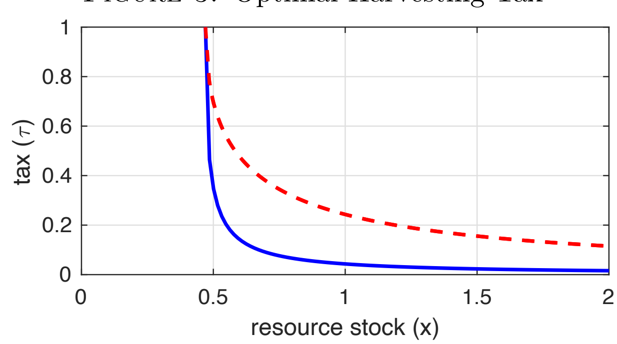
The derivation of the optimal tax rate as well the proof of its comparative statics results stated in the next Proposition are provided in the Appendix.

Proposition 4. *The optimal tax rate τ^* that corrects the self-control problem of harvesting a renewable resource too excessively increases in the degree of limited self-control ω and declines in the present stock of the resource x.*

These comparative statics are intuitive. A higher tax reduces the net return per harvested unit and thus the incentive to harvest too much. Facing relatively low returns of continuing fishing, impatient fishermen call it a day earlier and return with the long-run optimal catch size. When the fish stock is low, the harvesting rate should be reduced more and overfishing due to limited self-control is a greater threat. Thus, the optimal tax is declining in x.

Finally, we illustrate the optimal tax with a numerical example. Benchmark parameters are shown below Figure 3. The blue line shows results for $\Omega = 0.1$ (small self-control problem). The red line shows results for $\Omega = 0.9$ (large self-control problem). For small self-control problems the optimal tax can be smaller. With rising resource stock, tax rates converge to zero. With declining resource stock tax rates converge to unity at a finite $\tilde{x}$ (here, at about $x = 0.47$). This means that, if the initial resource stock is too far below its socially optimal steady state, there exists no feasible tax that establishes instantly the long-run optimum. This does not mean that tax policy is futile. A reasonable tax policy gradually lowers harvesting and improves the resource stock until a situation is reached where the long-run optimum is implementable. This can best be seen for the case of $\tau = 1$, which effectively means a fishing ban until the social optimum becomes implementable.

Figure 3. Optimal Harvesting Tax



Parameters: $\alpha = p = \kappa = 1$, $r = 0.5$, $\rho = 0.03$. Blue line $\Omega = 0.1$; red line: $\Omega = 0.9$

3. Limited Self-control and the Tragedy of the Commons

We next generalize the theory towards many agents harvesting a common pool resource. Specifically there are n symmetric agents (fishermen) who face the same given resource price p, the same cost function, and thus the same profit function, denoted by $\pi_i = [p(1-\tau)-\alpha h_i/x]h_i$ for agent $i = 1, \ldots, n$. This dynamic formulation of the tragedy of the commons (Hardin, 1986) is know as a fish war model (Levhari and Mirman, 1980). Open loop strategies are easily obtained. They coincide with the solution of the basic model. Here we focus on time-consistent feedback (Markovian) harvesting strategies, $h_i = h_i(x)$. These kind of differential games lead to partial differential equations and are generally not accessible analytically (see Long, 2010; Dockner et al., 2000). It turns out, however, that we can solve the problem in closed form when we linearize the resource growth equation. We thus assume natural growth of the resource according to $g(x) = \nu - \delta x$, $\nu > 0$, $\delta > 0$. Without harvesting, the resource stock converges to the level ν/δ.

With n non-cooperating agents (fishermen), the evolution of the common pool resource stock evolves according to:

$$\dot{x} = \nu - \delta x - \sum_{j=1}^{n} h_j. \tag{10}$$

The problem of excessive exploitation of the common pool resource is further aggravated by the fact that agents have only limited self-control. Specifically agent i maximizes

$$V_i = \int_0^\infty \mathrm{e}^{-\rho t} \left\{\pi_i(x, h_i) - \omega \left[\pi_i^s(x) - \pi_i(x, h_i)\right]\right\} \mathrm{d}t \tag{11}$$

subject to (10). Inserting the definition of profits and the profits desired by the short-run self, which are the same as for the unitary model of Section 2, the Hamiltonian for this problem is

$$\mathcal{H}_i = (1+\omega)\left[p(1-\tau)h_i - \frac{\alpha h_i^2}{x}\right] - \omega\left[\frac{p^2(1-\tau)^2}{4\alpha}\right]x + \lambda_i\left[\nu - \delta x - \sum_{j=1}^{n} h_j\right], \qquad i = 1, \ldots n.$$

The first order condition for h_i and the associated costate equation are

$$(1+\omega)\left[p(1-\tau) - \frac{2\alpha h_i}{x}\right] - \lambda_i = 0 \tag{12}$$

$$(1+\omega)\frac{\alpha h_i^2}{x^2} - \omega\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + \lambda_i\left[-\delta - \sum_{j=1, j\neq i}^{n} h_j'\right] = -\dot{\lambda}_i + \lambda_i \rho. \tag{13}$$

For the solution, we guess and verify that linear Markov strategies are a solution. Because of symmetry, all agents play the same strategy $h_i(x) = bx$, $i = 1, \ldots, n$, in which b is the undetermined coefficient. We thus have $h_i'(x) = b$ and $\lambda_i = (1+\omega)\left[p(1-\tau) - 2\alpha b\right]$ from (12), implying that λ_i stays constant. Using this information, (13) simplifies to

$$0 = \alpha b^2 - \frac{\omega}{1+\omega}\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + \left[p(1-\tau) - 2\alpha b\right]\left[-\delta - \rho - (n-1)b\right]. \tag{14}$$

Since, by definition, $b \in [0,1]$, this condition has the unique solution

$$b^* = \frac{-2\alpha(\delta+\rho) - p(1-\tau)(n-1) + \sqrt{\Lambda}}{2\alpha(2n-1)}, \quad \Lambda \equiv [2\alpha(\delta+\rho) + np(1-\tau)]^2 - \frac{(2n-1)p^2(1-\tau)^2}{1+\omega}. \tag{15}$$

System dynamics are fully described by (10) with $h_j(x) = b^* x$, and the initial stock $x(0)$. They lead to the steady state

$$x^* = \frac{\nu}{\delta + nb^*}. \tag{16}$$

Convergence towards the steady state fulfills the transversality condition $\lim_{t\to\infty} \lambda_i(t)x(t)\mathrm{e}^{-\rho t} = 0$. Notice that due to the linear strategy and the linear equation of motion for resource growth, extinction is excluded and a positive steady state always exists. The level of steady state resources, however can be reduced severely by limited self-control of the agents. From (15) follows immediately $\partial b/\partial\omega > 0$, on and off the steady state, and thus from (16), $\partial x^*/\partial\omega < 0$.

The impact of self-control on harvest depends on the degree of competition competition (number of competing agents) n:

Proposition 5. *Increasing competition weakens the impact of self-control problems.*

The proof inspects the derivative

$$\frac{\partial^2 b^*}{\partial\omega\partial n} = -\frac{p^3(\tau-1)^3\left(-2\alpha(\delta+\rho) - \frac{p(1-\tau)(n-1+n\omega)}{1+\omega}\right)}{4\alpha(1+\omega)^2\Lambda^{\frac{3}{2}}} < 0$$

This perhaps unexpected result indicates that as more groups compete about resource extraction, individual self-control problems become relatively less important. Instead, competition contributes directly to an increasing degree to overexploitation of the resource.

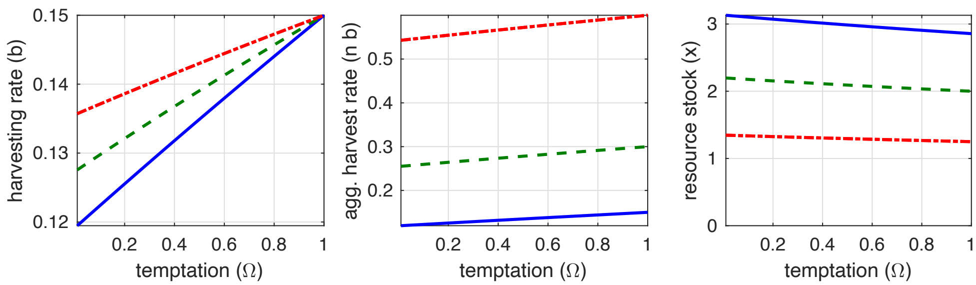
Proposition 6. *Increasing competition has direct impact on harvesting through the business stealing effect such that*

$$\frac{\partial b^*}{\partial n} > 0. \tag{17}$$

The proof is given in the Appendix.

Figure 4 illustrates these results with a numerical example. Blue solid lines represent the special case of only one agent harvesting (no competition). Green dashed lines reflect the case of two competing agents, and red dash-dotted lines the case of four agents. As for the basic model, the harvesting rate increases with rising self-control problems (increasing ω) As the number of agents increases, each agent harvests more but a greater value of Ω has a smaller marginal effect on the harvesting rate. For $\Omega \to 1$ all harvesting strategies converge to $b^* = 0.15$. This solution is intuitive since $\Omega \to 1$ represents the case where temptation utility becomes infinite. With infinite temptation, only instantaneous profits matter and these are independent from the degree of competition. The myopic solution is to harvest at rate $b = p(1-\tau)/(2\alpha)$, as shown in the Appendix, which is 0.15 for the numerical example.

Figure 4. Harvesting by Competing Agents with Limited Self-Control



Parameters: $\alpha = \nu = 1$, $p = 0.1$, $\delta = 0.05$, $\rho = 0.03$. Blue (solid) lines $n = 1$; green (dashed) lines: $n = 2$ red (dash-dotted) lines: $n = 4$.

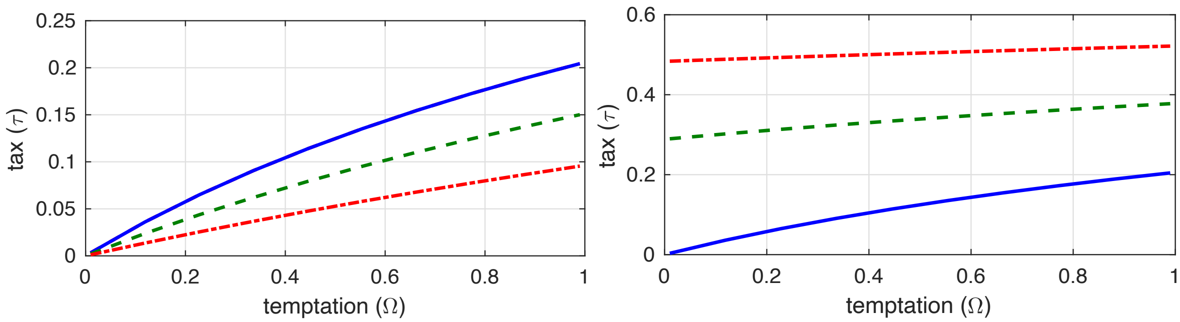
Next, we investigate taxation. We differentiate between two cases. In the first case the tax remedies only extensive harvesting due to limited self-control, as for the basic model. Results for the example from Figure 4 and different degrees of temptation are shown in Figure 5 on the left-hand side. The optimal tax increases in Ω and, perhaps surprisingly, it declines in the number of competing agents for given Ω. This result reflects the fact that if there is another externality at work due to missing property rights, which increases in the number of competing groups, self-control problems play a relatively smaller role.

To investigate this feature more closely, we compute taxes that remedy extensive harvesting due to limited self-control *and* due to competition on the commons, i.e. taxes that establish the social optimum. This means that taxes implement the solution for $\omega = 0$ and $n = 1$ (for $n = 1$ property rights are respected since there is only one resource owner). These solutions are shown in the panel on the right-hand side of Figure 5. For comparison, the blue line is the same in both panels. For the numerical example, a drastic tax is needed to internalize costs due to missing property rights. For large n, this tax dwarfs the tax needed to internalize self-control problems, and a larger Ω leads to only a small surcharge on the tax rate, as shown by the almost flat slope of the green curve, for $n = 4$.

We can also obtain an analytical solution for the optimal tax rate τ^*. It solves the equation $b_{\omega=0,\tau=0,n=1} = b_{\omega>0,\tau>0,n>1}$, which is obtained from (15). The solution, however, is several lines long and hard to assess intuitively. The following Proposition provides the comparative statics for the optimal tax rate. The proof is given in the Appendix.

PROPOSITION 7. *The optimal tax rate τ^* increases in the severity of self-control ω and in the number of competing agents n.*

FIGURE 5. Optimal Harvesting Tax in the Commons



Left-hand-side: tax rate that eliminates suboptimal harvesting due to self-control problems. Right-hand side: tax rate that eliminates suboptimal harvesting due to self-control problems and non-cooperative use of the common resource (i.e. tax rate establishes the social optimum). Parameters as for Figure 4. Blue (solid) lines $n = 1$; green (dashed) lines: $n = 2$ red (dash-dotted) lines: $n = 4$.

Finally, we investigate competing agents with different degrees of self-control. To simplify, we focus on two agents, indexed by 1 and 2 with self-control parameter ω_1 and ω_2 and temptation parameter Ω_1 and Ω_2. All other parameters are shared. As result, ω is now indexed by $i, j = 1, 2$ and the first order condition and costate equation are modified to:

$$(1+\omega_i)\left[p(1-\tau) - \frac{2\alpha h_i}{x}\right] - \lambda_i = 0 \tag{18}$$

$$(1+\omega_i)\frac{\alpha h_i^2}{x^2} - \omega_i\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + \lambda_i\left[-\delta - h_j'\right] = -\dot{\lambda}_i + \lambda_i\rho. \tag{19}$$

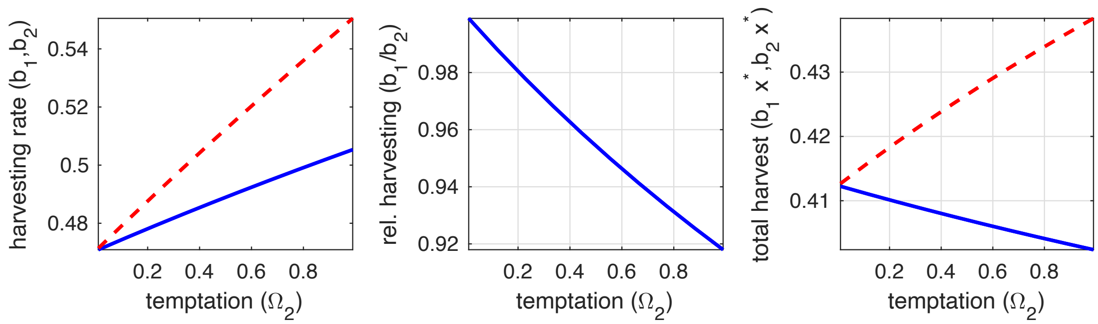
Assuming, as before, linear Markovian harvesting strategies $h_i(x) = b_i x$, but now abandoning the symmetry assumption, we obtain from (18) and (19)

$$0 = \alpha b_1^2 - \Omega_1\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + [p(1-\tau) - 2\alpha b_1][-\delta - \rho - b_2] \tag{20}$$

$$0 = \alpha b_2^2 - \Omega_2\left[\frac{p^2(1-\tau)^2}{4\alpha}\right] + [p(1-\tau) - 2\alpha b_2][-\delta - \rho - b_1] \tag{21}$$

This two dimensional system, can in principle be solved for the two unknowns b_1 and b_2. However, the non-linearity is not as easily resolved as in the symmetric case by applying the boundary condition $b_i \geq 0$. We thus proceed with a numerical exploration. Specifically, we are interested in how the presence of an impatient competitor affects behavior of the patient agent. For that we assume that agent 1 is perfectly rational without self-control problems ($\Omega_1 = 0$) and agent 2 faces a self-control problem of alternative severity ($\Omega_2 \in (0, 1]$). All other parameters are similar to the previous examples and summarized below Figure 6.

FIGURE 6. Harvesting by Agents with High and Low Self-Control



Parameters $\alpha = \nu = 1$, $p = 0.3$, $\delta = 0.05$, $\rho = 0.03$. Two agents: blue (solid) lines: high self-control ($\Omega_1 = 0$); red (dashed) lines: low self-control with varying Ω.

The results, shown in Figure 6, reveal that the self-control problem of agent 2 spills over to harvesting behavior of the fully rational agent. As Ω_2 rises, not only the impatient agent 2 harvests a greater share of the resource (dashed lines in the panel on the left-hand side) but also the fully rational agent 1 (solid lines). The intuition is that agent 1 takes into account that a conservative resource use becomes increasingly futile when the resource is shared with an agent who increasingly suffers from a self-control problem. A greater share of the resource not harvested by agent 1 will be harvested by agent 2. In other words, impatience of agent 2 reduces for agent 1 the return of investment in a greater resource stock. Since behavior spills not over completely, the limited restraint of agent 1 implies that the relative share of the resource harvested by agent 1 declines (b_1/b_2 in the center panel in Figure 6). As a result of the greater b's of both agents, the steady-state level of the resource declines ($x^* = \nu/(\delta + b1 + b2)$ declines). Interestingly, this implies that the total harvest of agent 1, $b_1 x^*$ declines while the harvest of agent 2 increases as self-control problems of agent 2 become larger (right panel in Figure 6). This means that agent 2 imposes the negative long-run consequences of his imperfect self-control on on agent 1.

4. Conclusion

In this paper, we introduced dual-self theory into models of renewable resource use. This modeling allowed us to discuss overexploitation and potential extinction of resources caused by present-biased preferences. The dual-self approach takes into account insights from psychology and neurology showing that different areas of the brain are occupied with short-run (impulsive) behavior and long-run (planned) behavior. The short-run self of harvesting agents desires to maximize instantaneous profits (utility) while the long-run self plans for the infinite future and takes repercussion of harvesting behavior on resource dynamics into account. Depending on the degree of self-control, actual behavior is partly driven by short-run desires. We first integrated present bias into the Schaefer-Gordon model and analyzed how self-control problems cause excessive harvesting and may jeopardize sustainability. We then computed optimal taxes that correct present bias and implement the long-run optimal harvesting behavior.

We extended the analysis towards several symmetric agents harvesting a common pool resource. We show that limited self-control amplifies the incentive for resource extraction and exacerbates the tragedy of the commons. We computed optimal tax policies that internalize the costs of overexploitation due to self-control problems as well as due to missing property rights. Finally, we showed that there are spillover effects of limited self-control in the sense that perfectly rational agents also behave less conservatively when they interact with agents with imperfect self-control. We also showed that, in groups of heterogenous harvesting agents, perfectly rational agents bear the costs of excessive harvesting behavior of agents with low self-control.

We deliberately designed models of renewable resource that are plausible and simple enough to be discussed analytically. Unfortunately, already mild extensions or generalizations are no longer accessible by analytic discussion. Numerical discussion of more complex models is of course possible and may generate further insights into harvesting behavior and sustainability when agents face self-control problems. The greatest value could perhaps be added to our study by empirical and experimental research on self-control problems in the context of renewable resource use.

Appendix

Steady State. Solving equation (1) for the steady state yields

$$x = 0 \quad \text{or} \quad b = r\left(1 - \frac{x}{\kappa}\right) \;\Leftrightarrow\; x = (r-b)\frac{\kappa}{r}. \tag{A.1}$$

Substituting

$$-\frac{rx}{\kappa} = b - r$$

into equation (5) leads to

$$\begin{aligned} & b^2 - 2b\,(2b - r - \rho) + \frac{p(1-\tau)}{\alpha}\left(2b - r - \rho - \frac{\omega}{1+\omega}\frac{p(1-\tau)}{4\alpha}\right) = 0 \\ \Leftrightarrow\; & b^2 - b\frac{2}{3}\left(r + \rho + \frac{p(1-\tau)}{\alpha}\right) + \frac{1}{3}\frac{p(1-\tau)}{\alpha}\left(2b - r - \rho - \frac{\omega}{1+\omega}\frac{p(1-\tau)}{4\alpha}\right) = 0 \end{aligned}$$

which is a quadratic equation in b with the solutions

$$b_{1,2} = \frac{2(\alpha(r+\rho) + p(1-\tau)) \pm \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + (4 - 3\frac{\omega}{1+\omega})p^2(1-\tau)^2}}{6\alpha}.$$

Proof of Proposition 1. The maximizing Hamiltonian is concave in x if and only if

$$\frac{\partial^2 \mathcal{H}}{\partial x^2} = -\frac{2r\lambda_i}{\kappa} < 0 \;\Leftrightarrow\; \lambda_i > 0.$$

From the first order condition (3) we obtain

$$\lambda_i = (1+\omega)\,(p(1-\tau) - 2\alpha b) > 0 \;\Leftrightarrow\; b < \frac{p(1-\tau)}{2\alpha}.$$

The first steady state value b_1^* is decreasing in the severity of self control $\Omega \in [0,1)$. Hence,

$$\begin{aligned} b_1^* &= \frac{2(\alpha(r+\rho) + p(1-\tau)) + \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + (4-3\Omega)p^2(1-\tau)^2}}{6\alpha} \\ &> \frac{2(\alpha(r+\rho) + p(1-\tau)) + \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + p^2(1-\tau)^2}}{6\alpha} =: b_{1,\min}. \end{aligned}$$

Furthermore, we have

$$b_{1,\min} \geq \frac{p(1-\tau)}{2\alpha} \;\Leftrightarrow\; 2(\alpha(r+\rho) + p(1-\tau)) + \sqrt{(2\alpha(r+\rho) - p(1-\tau))^2} \geq 3p(1-\tau).$$

This is equivalent to

$$\sqrt{(2\alpha(r+\rho) - p(1-\tau))^2} \geq p(1-\tau) - 2\alpha(r+\rho),$$

which is always true. Therefore, the maximizing Hamiltonian is convex in (x_2^*, b_2^*), which is therefore a local minimum.

The second steady state value b_2^* is monotonically increasing in the severity of self control $\Omega \in [0,1)$. Hence,

$$\begin{aligned} b_2^* &= \frac{2(\alpha(r+\rho) + p(1-\tau)) - \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + (4-3\Omega)p^2(1-\tau)^2}}{6\alpha} \\ &< \frac{2(\alpha(r+\rho) + p(1-\tau)) + \sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(1-\tau)(r+\rho) + p^2(1-\tau)^2}}{6\alpha} =: b_{2,\max}. \end{aligned}$$

Furthermore, we have

$$b_{2,\max} \leq \frac{p(1-\tau)}{2\alpha} \;\Leftrightarrow\; 2(\alpha(r+\rho)+p(1-\tau)) - \sqrt{(2\alpha(r+\rho)-p(1-\tau))^2} \leq 3p(1-\tau).$$

This is equivalent to

$$-\sqrt{(2\alpha(r+\rho)-p(1-\tau))^2} \leq p(1-\tau) - 2\alpha(r+\rho)$$

which is always true. Therefore, the maximizing Hamiltonian is concave in (x_2^*, b_2^*), which is therefore a local maximum.

Proof of Proposition 2. The partial derivatives of b^* are given by

$$\frac{\partial b^*}{\partial \Omega} = \frac{p^2(1-\tau)^2}{4\alpha\sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(r+\rho)(1-\tau) + p^2(1-\tau)^2(4-3\Omega)}} > 0$$

$$\frac{\partial b^*}{\partial r} = \frac{\partial b^*}{\partial \rho} = \frac{1}{3}\left(1 + \frac{p(1-\tau) - 2\alpha(r+\rho)}{\sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(r+\rho)(1-\tau) + p^2(1-\tau)^2(4-3\Omega)}}\right) > 0$$

$$\Leftrightarrow 2\alpha(r+\rho) - p(1-\tau) < \sqrt{[2\alpha(r+\rho)-p(1-\tau)]^2 + 3(1-\Omega)p^2(1-\tau)^2}$$

$$\frac{\partial b^*}{\partial \alpha} = \frac{p(1-\tau)}{6\alpha^2}\left[p(1-\tau)(4-3\Omega) - 2\left(\alpha(r+\rho) + \sqrt{[2\alpha(r+\rho)-p(1-\tau)]^2 + 3(1-\Omega)p^2(1-\tau)^2}\right)\right] < 0$$

$$\frac{\partial b^*}{\partial \tau} = \frac{p}{6\alpha}\left[-2 + \frac{-2\alpha(r+\rho) + p(1-\tau)(4-3\Omega)}{\sqrt{[2\alpha(r+\rho)-p(1-\tau)]^2 + 3(1-\Omega)p^2(1-\tau)^2}}\right] < 0$$

$$\Leftrightarrow \Lambda := -2 + \frac{-2\alpha(r+\rho) + p(1-\tau)(4-3\Omega)}{\sqrt{[2\alpha(r+\rho)-p(1-\tau)]^2 + 3(1-\Omega)p^2(1-\tau)^2}} < 0.$$

It holds

$$\frac{\partial \Lambda}{\partial (r+\rho)} = -\frac{12p(r+\rho)\alpha^2(1-\tau)(1-\Omega)}{[(2\alpha(r+\rho)-p(1-\tau))^2 + (3-3\Omega)p^2(1-\tau)^2]^{\frac{3}{2}}} < 0.$$

Hence, Λ is maximized for $r+\rho=0$, and, therefore

$$\frac{6\alpha}{p}\frac{\partial b^*}{\partial \tau} < -2 + \frac{p(1-\tau)(4-3\Omega)}{\sqrt{(4-3\Omega)p^2(1-\tau)^2}} = -2 + \sqrt{4-3\Omega} \leq 0.$$

Using a similar argument it is trivial to see that

$$\frac{\partial b^*}{\partial p} = \frac{1}{6\alpha}\left(2(1-\tau) + \frac{(1-\tau)\left(2\alpha(r+\rho) - p(1-\tau)(4-3\Omega)\right)}{\sqrt{4\alpha^2(r+\rho)^2 - 4\alpha p(r+\rho)(1-\tau) + p^2(1-\tau)^2(4-3\Omega)}}\right) > 0.$$

Proof of Proposition 3.

$$\frac{\partial^2 b^*}{\partial \Omega \partial \rho} = \frac{-p^2(1-\tau)^2(2\alpha(r+\rho)-p(1-\tau))}{2\left\{4\alpha^2(r+\rho)^2 - 4p\alpha(r+\rho)(1-\tau) - p^2(1-\tau)^2(-4+3\Omega)\right\}^{3/2}} > 0 \quad \Leftrightarrow \quad \frac{p(1-\tau)}{2\alpha} > r+\rho$$

$$\frac{\partial^2 b^*}{\partial \Omega \partial p} = -\frac{p(1-\tau)^2\left\{-8\alpha^2(r+\rho)^2 + 6p\alpha(r+\rho)(1-\tau) - p^2(1-\tau)^2(4-3\Omega)\right\}}{4\alpha\left\{4\alpha^2(r+\rho)^2 - 4p\alpha(r+\rho)(1-\tau) + p^2(1-\tau)^2(4-3\Omega)\right\}^{3/2}} > 0$$

$$\Leftrightarrow \quad \frac{p(1-\tau)}{2\alpha} \leq r+\rho; \quad \text{or} \quad \frac{p(1-\tau)}{2\alpha} \geq 2(r+\rho);$$

$$\text{or} \quad r+\rho \leq \frac{p(1-\tau)}{2\alpha} \leq 2(r+\rho) \quad \text{and} \quad \Omega < \frac{2}{3}\left\{2+\frac{\alpha(r+\rho)(4\alpha(r+\rho)-3p(1-\tau))}{p^2(1-\tau)^2}\right\}$$

$$\frac{\partial^2 b^*}{\partial\Omega\partial r} = -\frac{p^2(2\alpha(r+\rho)-p(1-\tau))(1-\tau)^2}{2\left\{4\alpha^2(r+\rho)^2-4p\alpha(r+\rho)(1-\tau)+p^2(1-\tau)^2(4-3\Omega)\right\}^{3/2}} > 0 \quad \Leftrightarrow \quad \frac{p(1-\tau)}{2\alpha} > r+\rho$$

$$\frac{\partial^2 b^*}{\partial\Omega\partial\alpha} = -\frac{\partial^2 b^*}{\partial\Omega\partial p}$$

Optimal tax rate. To calculate the optimal tax rate τ^* which internalizes the self control problem we have to solve the equation

$$\dot{b}_{|\tau>0,\omega>0} = \dot{b}_{|\tau=0,\omega=0}$$

for τ. This leads to

$$b^2-2b\left[r-\rho-\frac{2rx}{\kappa}\right]+\frac{p(1-\tau)}{\alpha}\left[r-\rho-\frac{2rx}{\kappa}-\frac{\omega}{1+\omega}\frac{p(1-\tau)}{4\alpha}\right] = b^2-2b\left[r-\rho-\frac{2rx}{\kappa}\right]+\frac{p}{\alpha}\left[r-\rho-\frac{2rx}{\kappa}\right]$$

$$\Leftrightarrow \frac{p\tau}{\alpha}\left[r-\rho-\frac{2rx}{\kappa}\right]+\frac{p(1-\tau}{\alpha}\frac{\omega}{1+\omega}\frac{p(1-\tau)}{4\alpha} = 0$$

$$\Leftrightarrow \tau\underbrace{\left[r-\rho-\frac{2rx}{\kappa}\right]}_{=:B<0}+(1-\tau)^2\underbrace{\frac{\omega}{1+\omega}\frac{p}{4\alpha}}_{=:A>0} = 0$$

which results in the two potential optimal tax rates

$$\tau_{1,2} = \frac{2A-B\pm\sqrt{B(B-4A)}}{2A}.$$

Since $A>0$ and $B<0$, it is easy to see that $\tau_1>1$ and $\tau_2\in(0,1)$. Therefore,

$$\tau^* = \tau_2 = \frac{2A-B-\sqrt{B(B-4A)}}{2A}$$

is the only feasible optimal tax rate.

Proof of Proposition 4. It holds

$$\frac{\partial\tau^*}{\partial A} = \frac{B(-2A+B+\sqrt{B(-4A+B)})}{2A^2\sqrt{B(-4A+B)}} > 0$$

$$\frac{\partial\tau^*}{\partial B} = -\frac{1}{2A}\left(1+\frac{-2A+B}{\sqrt{B(-4A+B)}}\right) > 0.$$

Together with

$$\frac{\partial A}{\partial\omega} > 0 \quad \text{and} \quad \frac{\partial B}{\partial\omega} = 0$$

$$\frac{\partial A}{\partial x} < 0 \quad \text{and} \quad \frac{\partial B}{\partial x} = 0$$

this leads to

$$\frac{\partial\tau^*}{\partial\omega} > 0 \quad \text{and} \quad \frac{\partial\tau^*}{\partial x} < 0.$$

Proof of Proposition 6. It holds

$$\frac{\partial b^*}{\partial n} = \frac{1}{2\alpha(2n-1)^2}\Big\{-2\left[-2\alpha(\delta+\rho)+p(1-\tau)(n-1)+\sqrt{\Lambda}\right] + (2n-1)\left[p(1-\tau)+\frac{p(1-\tau)\left(2\alpha(\delta+\rho)(1+\omega)+p(1-\tau)(n-1+n\omega)\right)}{(1+\omega)\sqrt{\Lambda}}\right]\Big\}$$

with

$$\Lambda := [2\alpha(\delta+\rho)+np(1-\tau)]^2 - \frac{(2n-1)p^2(1-\tau)^2}{1+\omega}.$$

We define $\tilde{p} := (1-\tau)p$ and $\tilde{\delta} := 2\alpha(\delta+\rho)$ to obtain

$$\frac{\partial b^*}{\partial n} > 0 \;\Leftrightarrow\; -2\left[-\tilde{\delta}+(n-1)\tilde{p}+\sqrt{\Lambda}\right]+(2n-1)\left\{\tilde{p}+\frac{\tilde{p}\left[\tilde{\delta}(1+\omega)+\tilde{p}(n-1+n\omega)\right]}{(1+\omega)\sqrt{\Lambda}}\right\} > 0.$$

This is equivalent to

$$\begin{aligned}
&2\tilde{\delta}+\tilde{p}-2\sqrt{\Lambda}+\frac{2n-1}{(1+\omega)\sqrt{\Lambda}}\left[\tilde{p}\tilde{\delta}(1+\omega)+\tilde{p}^2n(1+\omega)-\tilde{p}^2\right] > 0\\
\Leftrightarrow\;& (2\tilde{\delta}+\tilde{p})\sqrt{\Lambda}-2(\tilde{\delta}+n\tilde{p})^2+2\frac{(2n-1)\tilde{p}^2}{1+\omega}+(2n-1)(\tilde{p}\tilde{\delta}-n\tilde{p}^2)-\frac{2n-1}{1+\omega}\tilde{p}^2 > 0\\
\Leftrightarrow\;& (2\tilde{\delta}+\tilde{p})\sqrt{\Lambda}-(2\tilde{\delta}+\tilde{p})(\tilde{\delta}+n\tilde{p})+\frac{(2n-1)\tilde{p}^2}{1+\omega} > 0\\
\Leftrightarrow\;& (2\tilde{\delta}+\tilde{p})\left[\sqrt{\Lambda}-(\tilde{\delta}+n\tilde{p})\right]+\frac{(2n-1)\tilde{p}^2}{1+\omega} > 0\\
\Leftrightarrow\;& (2\tilde{\delta}+\tilde{p})\left[\Lambda-(\tilde{\delta}+n\tilde{p})^2\right]+\frac{(2n-1)\tilde{p}^2}{1+\omega}\left[\sqrt{\Lambda}+(\tilde{\delta}+n\tilde{p})\right] > 0\\
\Leftrightarrow\;& (2\tilde{\delta}+\tilde{p})\frac{-(2n-1)\tilde{p}^2}{1+\omega}+\frac{(2n-1)\tilde{p}^2}{1+\omega}\left[\sqrt{\Lambda}+(\tilde{\delta}+n\tilde{p})\right] > 0\\
\Leftrightarrow\;& (n-1)\tilde{p}-\tilde{\delta}+\sqrt{\Lambda} > 0.
\end{aligned}$$

The latter is true as $b^* > 0$.

Proof of $\lim_{\omega\to\infty} b^* = p(1-\tau)/2\alpha$**.**

$$\begin{aligned}
\lim_{\omega\to\infty} b^* &= \lim_{\omega\to\infty}\frac{1}{2\alpha(2n-1)}\Big[-2\alpha(\delta+\rho)+p(n-1+\tau(1-n)) + \sqrt{(2\alpha(\delta+\rho)+np(1-\tau))^2-\frac{(2n-1)p^2(1-\tau)^2}{1+\omega}}\Big]\\
&= \frac{1}{2\alpha(2n-1)}\left[-2\alpha(\delta+\rho)+p(n-1+\tau(1-n))+\sqrt{(2\alpha(\delta+\rho)+np(1-\tau))^2}\right]\\
&= \frac{p(1-\tau)}{2\alpha}.
\end{aligned}$$

Proof of Proposition 7. The equation

$$b_{\omega=0,\tau=0,n=1} = b_{\omega>0,\tau>0,n>1}$$

yields an implicit function

$$F := \delta + \rho - \frac{\sqrt{\alpha(\delta+\rho)(p+\alpha(\delta+\rho))}}{\alpha} + \frac{-2\alpha(\delta+\rho) + p(n-1+\tau(1-n)) + \sqrt{\Lambda}}{2\alpha(2n-1)} = 0.$$

The partial derivatives of F with respect to τ, n and ω are

$$\frac{\partial F}{\partial \tau} = \frac{1}{2\alpha(2n-1)}\left[p(1-n) + \frac{p(p(\tau-1)(n(n\omega+n-2)+1) - 2\alpha n(\omega+1)(\delta+\rho))}{(1+\omega)\sqrt{\Lambda}}\right] < 0$$

$$\frac{\partial F}{\partial \omega} = \frac{p^2(1-\tau)^2}{4\alpha(\omega+1)^2\sqrt{\Lambda}} > 0$$

$$\frac{\partial F}{\partial n} = \frac{\partial b^*}{\partial n} > 0.$$

The Implicit Function Theorem leads to

$$\frac{\partial \tau}{\partial \omega} = -\frac{\frac{\partial F}{\partial \omega}}{\frac{\partial F}{\partial \tau}} > 0 \qquad \text{and} \qquad \frac{\partial \tau}{\partial n} = -\frac{\frac{\partial F}{\partial n}}{\frac{\partial F}{\partial \tau}} > 0.$$

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