

# EVALUATION OF VARIABLE SIZE SAMPLING PLOTS FOR MONITORING OF FOREST CONDITION

## HODNOTENIE VARIABILNE VEĽKÝCH SKUSNÝCH PLÔCH PRE MONITOROVANIE STAVU LESA

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**ABSTRACT:** Two different types of estimators for per ha characteristics and proportions, i. e. classical ratio estimators for cluster sampling and ppz-estimators, are discussed with respect to their statistical properties in the case of systematically selected sample plots of variable size. A case study illuminates practical consequences of neglecting variable selection probabilities in monitoring volume, stem number and defoliation. Particularly, the target values volume, stem number, needle/leaf loss and number of trees in a damage class must be transformed plotwise into per ha values.

sample plots of variable size; ratio estimator; cluster sampling; point sampling; forest condition monitoring; needle/leaf loss

**ABSTRAKT:** V práci sa rozoberajú dva rozdielne typy odhadov pre hektárové charakteristiky a pre relatívne podiely (klasický podielový odhad pri skupinovom výbere a ppz-odhad) so zameraním na ich biometrické vlastnosti v prípadoch, keď skusné plochy sa vyberajú systematicky a majú variabilnú veľkosť. Na výsledkoch špeciálneho pokusu sa ilustrujú praktické dôsledky vznikajúce zo zanedbania variabilných výberových pravdepodobností pri monitorovaní porastovej zásoby, počtu stromov a defoliácie. Dokazuje sa, že zistené údaje o zásobe, počte stromov, strate asimilačných orgánov a početnosti stromov v stupňoch defoliácie je potrebné na každej skusnej ploche pred ich konečným zhodnotením prepočítať na hektárové hodnoty.

variabilne veľké skusné plochy; odhad podielu; skupinový výber; bodový výber; monitoring stavu lesa; defoliácia

### INTRODUCTION

The size of sample plots plays an important role in all sampling techniques for forest inventory. It influences not only the needs of time and costs but also the precision of inventory results. Therefore, one aims always at an optimal choice of sample size, considering those antagonistic effects of increasing costs and decreasing sampling error. In Slovakia, numerous investigations stated that such an optimal sample plot should include 15–25 trees on an average (Šmelko, 1968). Thus, in younger stands with large stem numbers of e. g. 1000 trees per ha a plot size of about 200 m<sup>2</sup>, and in older stands with about 200 trees per ha, a plot size of 1000 m<sup>2</sup> must be chosen. Normally, single stands are much more homogeneous with respect to stem number, age mixtures and so on than larger forest areas. Therefore, higher effectiveness is expected in large scale inventories by attaching one optimal plot size to each stand, varying between stands dependent on the individual stem numbers per ha.

Together with the application of variable plot sizes the question of how to estimate usual population parameters of state and change and their sampling errors arises. Beyond the variability of target variables like volume, stem number or leaf loss, the variability of sample plot areas must also be considered. At a first glance classical ratio estimators seem to be the answer. This problem was part of a joined project supported by *Zusammenarbeit in der Agrarforschung Bundesrepublik Deutschland/Slowakische Republik*. The paper deals with the statistical background of concurring estimators and their application in a forest monitoring project in northern Slovakia.

### DATA BASE

In Oravská Polhora, a region in northern Slovakia, where the forests suffer under high immissions,  $n = 32$  permanent sample plots were selected following a square grid of 1 km mesh width. The plots are circles of dif-

ferent sizes from 100 m<sup>2</sup> to 1000 m<sup>2</sup>, each containing about 23 trees on an average. They are located mainly in even aged stands of spruce, with stand ages varying from 30 to 150 years, stocking degrees of 0.5–0.9 and elevations of 720 m to 1350 m above sea level. The midpoints are permanently marked in order to facilitate repeated measurements of all quantitative and qualitative characteristics of the same individuals. Another advantage is the high correlation between consecutive measurements. In this paper we utilize measurements of 1991 and 1995, characterising volume and health condition of trees in sample plots (Tab. I).

The net of sample plots represents a typical systematic sampling design with variable plot size. In chapter 2 we discuss the appropriate estimators of totals and ra-

tios for that design, based on selection probabilities proportional to plot size (ppz). In chapter 3 the sample data are evaluated by both the simple random cluster sample estimators and the ppz-estimators, in order to exemplify the generally remarkable differences, which can be explained by the biases of the first of both types of estimators.

#### EVALUATION OF VARIABLE SIZE SAMPLE PLOTS: RANDOM CLUSTER SAMPLING VERSUS SYSTEMATIC POINT SAMPLING

Let  $y_i$  be the value of the target variable (volume, stem number, number of trees belonging to a certain

I. Characteristics of single sample plots of case study Orava [ $F_i$  – plot size,  $m_i$  – stem number,  $v_i$  – volume (m<sup>3</sup>),  $D_i$  – defoliation, mean needle loss per tree (%)]

Plot		1991			1995			Utilization			Mortality		
No.	$F_i$	$m_i$	$v_i$	$D_i$	$m_i$	$v_i$	$D_i$	$m_i$	$v_i$	$D_i$	$m_i$	$v_i$	$D_i$
1	500	22	10.2	67.0	22	10.4	67.9						
2	500	23	18.0	49.6	23	18.9	58.0						
3	500	22	28.9	18.9	22	30.2	49.8				2	1.6	55.0
4	300	21	9.9	32.0	19	9.8	43.2	2	11.1	40.0	2	0.4	77.5
5	300	27	9.9	4.8	22	9.5	18.4	5	1.7	16.0	2	0.2	25.0
6	900	26	67.6	42.5	26	71.2	49.6						
7	700	32	44.5	38.7	30	46.0	46.3	2	1.6	60.0			
8	300	25	12.5	25.8	24	14.1	33.1	1	0.1	100.0			
9	400	24	21.7	40.2	23	23.3	41.5	1	0.8	30.0	1	1.3	70.0
10	300	21	13.6	8.1	13	12.1	15.4	8	2.5	7.3			
11	200	20	4.1	16.0	20	5.5	14.5						
12	1000	27	51.0	51.7	25	51.1	67.4	2	4.2	55.0	4	7.0	47.5
13	700	28	40.6	31.1	27	43.6	31.7	1	1.5	20.0	2	0.9	82.5
14	300	26	9.9	34.8	26	11.2	36.9						
15	300	23	12.5	11.1	17	11.7	6.8	6	2.1	42.5			
16	300	22	10.1	33.8	22	11.9	41.6						
17	300	25	14.6	17.0	24	17.0	18.9	1	0.1	60.0			
18	700	30	31.2	36.8	29	33.3	38.4	1	1.8	40.0			
19	800	25	19.7	44.2	24	21.0	58.5	1	0.2	60.0	1	0.4	80.0
20	500	20	34.2	28.0	20	35.0	37.0				1	2.0	40.0
21	300	21	19.6	39.8	18	18.6	47.7	3	1.8	68.3	2	3.4	22.5
22	300	23	14.6	38.3	23	15.5	42.4						
23	100	21	1.6	0.0	17	2.4	0.0	4	0.1	0.0			
24	300	22	8.2	38.6	22	8.9	49.3						
25	600	24	13.2	33.1	22	13.4	47.0	2	0.9	57.5			
26	700	13	29.8	39.2	12	30.3	44.2	1	1.0	65.0			
27	400	26	17.6	31.7	26	19.1	25.4						
28	400	19	12.1	52.6	19	13.1	56.8						
29	500	25	28.1	37.0	25	29.2	41.0				2	0.8	90.0
30	800	23	48.9	51.9	21	46.8	45.2	2	2.8	90.0	1	0.5	75.0
31	700	26	25.4	43.8	24	25.4	43.5	2	1.3	60.0			
32	600	21	12.5	50.9	20	13.0	60.8	1	0.7	45.0			

damage class) at sample plot  $i$ ,  $F_i$  the plot area and  $n$  the sample size. Then the ratio estimator (*ratio of means*)

$$\hat{y}_{R, ha} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n F_i} \quad (1)$$

is known to be a consistent estimator of

$$y_{ha} = \frac{\sum_{i=1}^N y_i}{\sum_{i=1}^N F_i} \quad (2.1)$$

the true per ha value of the target variable  $y$ , if (i) the entire population to be inventoried is tessellated by all possible sample plots (i. e. nonoverlapping and exhaustive clusters) and (ii) the latter are selected with equal probabilities (Cochran, 1977). Both assumptions are usually not fulfilled in forest inventory practice, and, consequently, (2.1) will generally be biased. Instead, the sample plots (mostly circles) are selected as a point sample from an infinite universe of possible plot mid-points, and trees belonging to stands, where larger plot sizes are chosen, are selected with higher probability than those of smaller plots. The selection probability is given by  $p_i = F_i/F$ ,  $F$  denoting the total area to be inventoried. This remains also true if the sample points are selected by a randomly located (usually rectangular) sample grid. Hence, the unbiased ppz-estimator (*mean of ratios*)

$$\hat{y}_{ha} = \frac{1}{F} \cdot \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{F_i} = \frac{1}{n} \sum_{i=1}^n y_{i, ha} \quad (2.2)$$

( $y_{i, ha} = y_i/F_i$ ) is the appropriate one. For a more detailed discussion, proofs and an additional simulation study see Saborowski, Šmelko (1998), for a sound mathematical background of point sampling in forest inventory Mandallaz (1991).

Expectation true estimators for the variance of  $\hat{y}_{ha}$  do not exist for systematic sampling. We use the variance estimator for simple ppz-sampling

$$V(\hat{y}_{ha}) = \frac{1}{n} \cdot \frac{1}{n-1} \sum_{i=1}^n (y_{i, ha} - \hat{y}_{ha})^2 \quad (2.3)$$

which was shown to be a conservative variance estimator in many applications of systematic sampling, and estimate the sampling error  $\sigma_{y_{ha}}^2$  by  $s_{y_{ha}}^2 = \sqrt{V(\hat{y}_{ha})}$ .

The principle of using individual per ha values of target variables for each sample plot, argued for above, can easily be transferred to more complex, composed estimators. If e.g. the parameter to be estimated is a ra-

tio, like the proportion  $p = k/m$  of trees belonging to a certain damage class ( $k$ : number of trees of a class;  $m$ : total number of trees), the totals  $k$  and  $m$  are estimated separately by unbiased ppz-estimators according to (2.2). This yields the ratio estimator (see also Saborowski et al., 1998)

$$\hat{p} = \frac{\hat{k}}{\hat{m}} = \frac{F \cdot \frac{1}{n} \sum_{i=1}^n \frac{k_i}{F_i}}{F \cdot \frac{1}{n} \sum_{i=1}^n \frac{m_i}{F_i}} = \frac{\sum_{i=1}^n k_{i, ha}}{\sum_{i=1}^n m_{i, ha}} \quad (2.4)$$

based on numbers per ha for each plot, and the common variance estimator

$$V(\hat{p}) = \frac{1}{\bar{m}_{ha}^2} \cdot \frac{1}{n(n-1)} \sum_{i=1}^n (k_{i, ha} - \hat{p}m_{i, ha})^2 \quad (2.5)$$

where  $\bar{m}_{ha} = \sum_{i=1}^n m_{i, ha} / n$ .

A similar problem arises with estimating the mean percentage of needle/leaf loss per tree. Let  $y_{ij}$  be the needle/leaf loss percentage of tree  $j$  in sample plot  $i$  consisting of  $m_i$  trees and

$$y_i = \sum_{j=1}^{m_i} y_{ij}$$

its plot total. Then, in the case of simple random sampling of clusters, the ratio estimator

$$\hat{\bar{y}}_R = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n m_i} = \frac{\sum_{i=1}^n m_i \bar{y}_i}{\sum_{i=1}^n m_i} \quad (2.6)$$

(Cochran, 1977) is recommended. It can be interpreted as a ratio of two estimators, for the total of needle/leaf loss percentages and the total of trees in the inventory region, respectively. Therefore, both must be replaced again by the according ppz-estimators if the sample plots are selected by a systematic grid, as mentioned above. Thus, in that case we use

$$\hat{\bar{y}}_{ppz} = \frac{\sum_{i=1}^n y_{i, ha}}{\sum_{i=1}^n m_{i, ha}} = \frac{\sum_{i=1}^n m_{i, ha} \bar{y}_i}{\sum_{i=1}^n m_{i, ha}} \quad (2.7)$$

and the variance estimator (2.5) with  $y_{i,ha}$  instead of  $k_{i,ha}$ , and  $\hat{y}_{ppz}$  replacing  $\hat{p}$

**ESTIMATION OF THE ACTUAL STATE  
AND CHANGE OF VOLUME AND HEALTH  
OF TREES**

**Estimation of stem number and volume**

Firstly, this chapter deals with the estimation of stem number and volume per ha in the inventoried forest region. Following the arguments above (chapter 2), the plot characteristics volume ( $v_i$ ) and stem number ( $m_i$ ) must be related to the plot size  $F_i$ , yielding the transformed variables  $v_{i,ha} = v_i/F_i$  and  $m_{i,ha} = m_i/F_i$ , which must be substituted for  $y_{i,ha}$  in formulas (2.2) and (2.3). The results for that mean of ratios estimator are denoted by A in Tab. II. For comparison, B denotes the according results for the ratio of means estimator given by (2.1) with  $y_i = v_i$  and  $y_i = m_i$  as well.

teristics at identical individuals. This follows e. g. from the variance formula

$$\sigma_{\Delta \hat{y}_{ha}}^2 = \sigma_{\hat{y}_{ha,1}}^2 + \sigma_{\hat{y}_{ha,2}}^2 - 2 \cdot r_{\hat{y}_{ha,1}; \hat{y}_{ha,2}} \cdot \sigma_{\hat{y}_{ha,1}} \cdot \sigma_{\hat{y}_{ha,2}} \quad (3.1)$$

for the difference of the per ha characteristic

$$\Delta \hat{y}_{ha} = \hat{y}_{ha}(t_2) - \hat{y}_{ha}(t_1) = \hat{y}_{ha,1} - \hat{y}_{ha,2} \quad (3.2)$$

A strong positive correlation can reduce the sampling error of the estimated difference remarkably. The statistical significance of a difference, that means of the change of a state variable is tested by the paired *t*-test

$$t = \frac{\hat{y}_{ha,2} - \hat{y}_{ha,1}}{S_{\Delta \hat{y}_{ha}}} \quad (3.3)$$

II. Estimation of stem number and standing volume per ha ( $\hat{y}_{ha}$ ) and their changes  $\Delta \hat{y}_{ha}$

Target var.	Method	1991		1995		Utilization	Mortality	Difference		Increment		
		$\hat{y}_{ha} \pm S_{\hat{y}_{ha}}$	$S_{\hat{y}_{ha}}^2$ (%)	$\hat{y}_{ha} \pm S_{\hat{y}_{ha}}$	$S_{\hat{y}_{ha}}^2$ (%)			$\hat{y}_{ha}$	$\hat{y}_{ha}$	$\Delta \hat{y}_{ha} \pm S_{\Delta \hat{y}_{ha}}$	<i>t</i> -test	$\Delta \hat{y}_{ha} \pm S_{\Delta \hat{y}_{ha}}$
Stem n. per ha	A	602.3 ± 61.3	10.2	555.8 ± 51.2	(9.2)	46.5	13.1	46.5 ± 15.8	2.93*			0.976
	B	485.8 ± 37.2	7.6	456.1 ± 34.2	(7.5)	29.7	12.9	29.7 ± 12.9	2.30*			0.938
Volume per ha	A	423.3 ± 27.9	6.6	443.5 ± 27.5	(6.2)	16.9	10.9	20.2 ± 7.5	2.68*	37.1 ± 7.5	4.92*	0.980
	B	453.9 ± 33.3	7.3	470.0 ± 34.1	(7.2)	22.8	20.5	16.1 ± 7.1	2.27*	38.9 ± 7.1	5.52*	0.978

Case study Orava,  $n = 32$ ,  $F_i = 100-1000 \text{ m}^2$

A - mean of ratios estimator

B - ratio of means estimator

The estimated sampling errors are rather large because of the low sample size. Method A gives lower volumes per ha (and also smaller standard errors) than B indicating a positive bias of B. The reverse is true for the stem numbers per ha. The coefficients of variation of the pure stem numbers ( $s_m\% = 15\%$  and  $18\%$ ) are much lower than those of the stem numbers per ha ( $s_{m,ha}\% = 57\%$  and  $52\%$ ). The correlations between plot areas  $F_i$  and stem numbers  $m_i$  are low ( $\hat{r}_{m,F} = 0.323$  and  $0.378$ ).

Obviously, plot sizes are well adapted to the stem numbers because they yield a low variability of stem numbers per plot and a weak linear dependence of stem numbers on plot sizes. Furthermore, the results exhibit remarkable differences between A and B, which cannot be neglected in forest inventories. The classical ratio estimator is a bad choice with a high risk of large biases.

Beyond assessments of the actual state of forests, monitoring also aims to detect changes of that state, i.e. differences in totals, means and ratios between two inventories at  $t_1$  and  $t_2$ . Application of permanent sample plots is generally expected to increase precision of such estimated differences by exploiting positive correlations between repeated measurements of tree charac-

with 31 degrees of freedom. This is necessary in order to avoid an overinterpretation of differences which can arise merely on account of sampling errors.

If we focus on volume, we have to distinguish between *change of standing volume*, estimated by the difference (3.2), and *volume increment*. Estimation of volume increment requires the additional consideration of the amount of timber utilization during a monitoring period ( $t_1, t_2$ ).

Tab. II shows significant changes of volume and stem numbers per ha at the 5% error level. Remarkable are the high correlations of volume and stem numbers per ha between 1991 and 1995. Again, the differences between A and B are not negligible.

**Estimation of percentage of trees in damage classes**

Another objective of the monitoring project was assessment of forest damage. The trees of a sample plot are classified into 5 damage classes 0, 1, 2, 3, 4 depending on their individual defoliation, the latter increasing from class 0 to class 4. Additional aggregated classes 2+3+4 and 3+4 are usually considered for evaluation. The proportion of trees belonging to each of those damage

classes should be estimated by  $\hat{p}$  according to (2.4) because of the varying selection probabilities for trees within plots of different sizes. Consequently, the change of proportions from 1991 to 1995 is estimated by

$$\Delta\hat{p} = \hat{p}(t_2) - \hat{p}(t_1) = \hat{p}_2 - \hat{p}_1 \quad (3.4)$$

the variance, by analogy with (3.1), by

$$s_{\Delta\hat{p}}^2 = s_{\hat{p}_1}^2 + s_{\hat{p}_2}^2 - 2 \cdot s_{\hat{p}_1\hat{p}_2} \quad (3.5)$$

where  $s_{\hat{p}_1\hat{p}_2}$  is the covariance estimator

$$s_{\hat{p}_1\hat{p}_2} = \frac{1}{m_{ha,1} \cdot m_{ha,2}} \cdot \frac{1}{n(n-1)} \cdot \sum_{i=1}^n (k_{i,ha,1} - \hat{p}_1 m_{i,ha,1})(k_{i,ha,2} - \hat{p}_2 m_{i,ha,2}) \quad (3.6)$$

(see Saborowski et al., 1998; Saborowski, 1990) and the  $s_{\hat{p}_1}^2, s_{\hat{p}_2}^2$  are the variance estimators according to (2.3).

The results of the forest damage study are reported in Tab. III. The letter A denotes results of (2.4) and (2.5) and B of the according estimators, where  $k_{i,ha}$  and  $m_{i,ha}$  are substituted by  $k_i$  and  $m_i$ . Method A gives lower percentages than B for all classes except class 0. The clearest difference of more than 10% for both years is found for the combined class 2+3+4. This coincides with the results in Saborowski et al. (1998) for the forest damage inventory in Lower Saxony. It can be explained by the higher selection probability for older trees having generally more considerable defoliations. This higher selection probability is not considered by method B. Estimation of change yields similar results for both methods. Correlations are high, but, in nearly all cases, lower than for volumes.

Estimation of changes (A) shows a clear shift of trees from classes 0 and 1 to classes 2, 3 and 4. The increments of proportions in classes 3, 4 and the combined classes as well as the decrement in class 0 are significant at an error level of 5%. This deterioration of health condition was not remarkably obscured by utilization because only 56% of trees were utilized in classes 2–4 and 30% in class 0, in class 0 mainly because of snow- and windbreak.

#### Estimation of the mean percentage of needle/leaf loss per tree

Sometimes, it is preferred to concentrate the results of a forest damage inventory in a one-dimensional statistic, the *mean needle/leaf loss* per tree, instead of the multivariate characteristic damage class. Therefore a higher resolution of defoliation assessment is necessary. The analyses of this paper are based on an assessment in 5%-steps. According to our arguments in chapter 2, we recommend (2.5) (A) as an estimator of mean needle/leaf loss per tree and compare the results with

III. Estimations of proportions of trees  $\hat{p}$  (%) in 5 damage classes and their changes  $\Delta\hat{p}$  (%)

a) proportions $\hat{p} \pm s_{\hat{p}}$ (%)				
Damage class	1991		1995	
	A	B	A	B
0	29.6 ± 9.4	17.8 ± 5.0	24.3 ± 9.1	13.6 ± 4.6
1	21.3 ± 3.7	21.9 ± 3.2	18.1 ± 3.4	17.7 ± 3.2
2	41.3 ± 6.5	49.9 ± 4.7	44.0 ± 6.2	51.7 ± 4.5
3	6.9 ± 1.7	9.3 ± 1.7	10.5 ± 2.2	13.2 ± 2.2
4	0.9 ± 0.4	1.1 ± 0.5	3.1 ± 0.8	3.8 ± 0.9
2+3+4	49.0 ± 7.5	60.3 ± 5.5	57.6 ± 7.9	68.7 ± 5.4
3+4	7.8 ± 1.9	10.4 ± 2.0	13.7 ± 2.6	17.0 ± 2.7

b) change of proportions $\Delta\hat{p} \pm s_{\Delta\hat{p}}$ (%) (* – significant change at 5% error level)						
Damage class	$\Delta\hat{p} \pm s_{\Delta\hat{p}}$		t-test		$r_{\hat{p}_1, \hat{p}_2}$	
	A	B	A	B	A	B
0	-5.3 ± 2.6	-4.2 ± 1.4	-2.07*	-3.00*	0.96	0.75
1	-3.2 ± 2.4	-4.2 ± 3.3	-1.32	-1.27	0.77	0.47
2	2.7 ± 4.2	1.8 ± 3.2	0.63	0.56	0.78	0.58
3	3.6 ± 1.5	3.9 ± 1.6	2.49*	2.44*	0.74	0.80
4	2.2 ± 0.8	2.7 ± 0.8	3.01*	3.37*	0.41	0.41
2+3+4	8.6 ± 3.7	8.4 ± 2.6	2.32*	3.23*	0.89	0.68
3+4	5.9 ± 1.7	6.6 ± 1.8	3.54*	3.67*	0.78	0.81

Case study Orava,  $n = 32$ ,  $F_i = 100-1000 \text{ m}^2$

A – estimation under consideration of variable selection probabilities proportional to  $F_i$

B – estimation without consideration of variable selection probabilities

those of the simple ratio estimator (2.4) (B), which does not consider the different selection probabilities. The variance of changes

$$\Delta\hat{y}_{ppz} = \hat{y}_{ppz,2} - \hat{y}_{ppz,1}$$

is estimated by analogy with the formulas in chapter 3.2.

Results are reported in Tab. IV for our inventory region in Orava. Method A exhibits an increase of mean needle/leaf loss per tree from  $28.5\% \pm 3.7\%$  to  $34.5\% \pm 4.2\%$  during the 4-year monitoring period. It is illuminating to recognize the contributions of utilization and mortality to the gross change. For utilized trees we estimated a slightly lower defoliation than for the entire population in 1991 and a much higher one for dead trees. The net change of 5.70%, excluding utilized trees, is not much less than the gross change (6.03%). The standard error of estimated change (1.65%) is much less than those of the state estimations in 1991 (3.7%) and 1995 (4.2%), due to the high correlation of 0.92 between defoliations in both years.

The differences between methods A and B are large in 1991 and 1995 but relatively low for the change estimations. The necessity of applying the statistically justified method A is obvious again.

IV. Estimation of mean needle/leaf loss (%) per tree and its change

Method	1991				1995			
	$\sum_i^n m_i$	$\bar{m}_{ha}$	$\hat{\bar{y}}$	$s_{\hat{\bar{y}}}$	$\sum_i^n m_i$	$\bar{m}_{ha}$	$\hat{\bar{y}}$	$s_{\hat{\bar{y}}}$
A		609	28.5	3.7		556	34.5	4.2
B	753		34.4	2.6	707		40.7	2.7

Method	Utilization			Mortality		
	$\sum_i^n m_i$	$\bar{m}_{ha}$	$\hat{\bar{y}}$	$\sum_i^n m_i$	$\bar{m}_{ha}$	$\hat{\bar{y}}$
A		53	25.4		14	55.3
B	46		37.9	21		53.8

Method	Change $\Delta\hat{\bar{y}}_{ppz}$ (A) and $\hat{\bar{y}}_R$ (B)					
	gross change			net change		
	$\Delta\hat{\bar{y}}$	$s_{\Delta\hat{\bar{y}}}$	t-test	$\Delta\hat{\bar{y}}$	t-test	$r_{91,95}^{\Delta\hat{\bar{y}}}$
A	6.03	1.65	3.65*	5.70	3.45*	0.920
B	6.32	1.05	6.01*	6.54	6.22*	0.923

Case study Orava,  $n = 32$ ,  $F_i = 100-1000 \text{ m}^2$

A – estimation under consideration of variable selection probabilities proportional to  $F_i$

B – estimation without consideration of variable selection probabilities

(\* – significant change at 5% error level)

CONCLUSION

Beyond the theoretical justification of method A, the results of the study in Orava exhibit clearly that the differences between A and B applied to sample data of a monitoring of production and health condition cannot be neglected in practice. It is very important to distinguish between sample plots of equal and unequal size and, consequently, between equal and unequal selection probabilities. The latter occur automatically if sample plots of variable size are selected by a systematic sam-

ple grid. In that case values of stem numbers and volumes must be transformed into per ha values in order to get unbiased, or in the case of proportions (relative frequencies of trees in a damage class) nearly unbiased, estimations. This is also true for estimating mean needle/leaf loss, where stem numbers per ha must be used instead of pure stem numbers in the usual cluster sampling estimator for clusters of unequal size. Otherwise, estimations and estimated sampling errors can have a remarkable bias.

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HODNOTENIE VARIABILNE VEĽKÝCH SKUSNÝCH PLŔCH PRE MONITOROVANIE STAVU LESA

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Veľkosť skusných plôch hrá veľmi významnú úlohu pri všetkých výberových spôsoboch inventarizácie lesa. Ovplyvňuje nielen pracovné náklady v teréne, ale aj variabilitu zisťovaných veličín na skusných plochách a tým aj presnosť výsledkov inventarizácie. Preto je všeobecnou snahou zvoliť si pri zohľadnení obidvoch protichodných tendencií (rastúcich nákladoch a klesajúcej výberovej chybe) optimálnu veľkosť skusných plôch. Na Slovensku bolo zásluhou rozsiahlych výskumov

(Š m e l k o, 1968) zistené, že za optimálnu čo do presnosti a hospodárnosti možno považovať takú skusnú plochu, ktorá obsahuje priemerne 15–25 stromov. To znamená, že v mladších a hustejších porastoch napr. s 1 000 stromami na 1 ha je optimálnou skusná plocha o výmere 200 m<sup>2</sup>, v starších a redších porastoch napr. s 200 stromami na 1 ha je to 1 000 m<sup>2</sup>. Uvedená úloha sa dá riešiť oveľa ľahšie pri porastovej inventarizácii ako pri veľkoplošných inventarizáciách. Jednotlivé porasty majú

spravidla homogénnejšiu vnútornú štruktúru (drevinovú skladbu, vek, počet stromov, zakmenenie ai.) ako veľké lesné oblasti. Pre jeden porast sa preto dá zvoliť jednotná optimálna veľkosť skusnej plochy. Pri veľkoplošnej inventarizácii a monitoringu sa naopak očakáva, že sa vyššia efektívnosť dosiahne použitím variabilnej, t.j. od porastu k porastu sa meniacej veľkosti skusných plôch.

Pri uplatnení variabilnej veľkosti skusných plôch vzniká však ďalší problém: podľa akých metodických postupov sa majú získané výberové údaje zhodnocovať, pretože tu okrem variability zisťovaného znaku na skusnej ploche treba zohľadniť ešte aj variabilitu veľkosti skusnej plochy.

Príspevok obsahuje riešenie tohto problému na základe nemecko-slovenskej spolupráce v rámci tzv. agrárneho výskumu (1996–1998). Podkladom sú údaje z výskumného objektu Orava, pre ktorý bola v r. 1991 založená sieť trvalých monitorovacích plôch (TMP) v lesnom hospodárskom celku Oravská Polhora v odstupoch 1 x 1 km. TMP sú kruhy o rôznej veľkosti 100–1 000 m<sup>2</sup> a zachytávajú prevažne smrekové porasty vo veku 30 až 150 rokov so zakmenením 0,5 až 0,9. Na každej TMP sa zistili všetky dôležité kvalitatívne a kvantitatívne veličiny stromov a v roku 1995 sa vykonalo opakované meranie. Pre príspevok sa použila údajová báza (tab. I) z 32 TMP o zásobe ( $v$ ), počte stromov ( $m$ ) a poškodení korún stromov (defoliácii –  $D$ ). Skutočnosťou, že meranie sa v r. 1991 i 1995 uskutočnilo na tom istom súbore stromov, získala sa možnosť zhodnotiť nielen stav uvedených veličín, ale aj ich zmenu počas monitorovacieho cyklu vrátane ťažby i mortality, a súčasne sa zabezpečila veľmi tesná korelácia medzi údajmi na začiatku a na konci monitorovania, čo má veľkú metodickú výhodu.

Pred vlastným zhodnotením uvedených údajov sa v príspevku osobitná pozornosť venuje teoretickému rozboru biometrických vlastností variabilne veľkých skusných plôch. Zavádza sa nasledovná symbolika:  $y_i$  – hodnota cieľovej veličiny (zásoba, počet stromov, početnosť stromov patriacich do určitého stupňa defoliácie) na  $i$ -tej skusnej ploche,  $F_i$  – výmera  $i$ -tej skusnej plochy,  $n$  – počet skusných plôch. Sieť TMP v objekte Orava sa kvalifikuje ako typický systematický výberový dizajn s variabilnou veľkosťou skusných plôch, t.j. taký, aký sa v inventarizácii a monitoringu stavu lesa najčastejšie používa v celej Európe. Vzhľadom na to, že stromy na skusných plochách nemajú rovnakú pravdepodobnosť dostať sa do výberu, táto sa rovná  $p_i = F_i/F$  ( $F$  – celková výmera inventarizovaného objektu) a je menšia pri malých a väčšia pri veľkých skusných plochách. To znamená, že na stanovenie priemernej hektárovej hodnoty  $\hat{y}_{ha}$  cieľového objektu sa musí použiť ppz-odhad („mean of ratios“ – priemer z podielov) podľa vzťahu (2.2) a jeho va-

riancia  $V(\hat{y}_{ha})$  sa určí podľa výrazu (2.3); stredná chyba odhadu  $\hat{y}_{ha}$  sa potom rovná druhej odmocnine z (2.3). Ako vidieť, do výpočtu nevchádzajú pôvodné hodnoty  $y_i$ , ale ich hektárové hodnoty  $y_{i;ha} = y_i/F_i$ . V prípade, že by sa takýto odhad urobil podľa modelu „ratio of means“, čiže ako podiel z priemerov podľa (2.1), ktorý platí len pre náhodný výber s rovnakými pravdepodobnosťami (Cochran, 1977), získali by sa systematicky vychýlené odhady. Tieto skutočnosti sú podrobnejšie zdokumentované a preverené v práci Saborowského, Šmelka (1998) a matematicky zdôvodnené v publikácii Mandallaza (1991). Podobná situácia nastáva aj pri odhade relatívneho podielu počtu stromov  $\hat{p}$  v jednotlivých stupňoch poškodenia, kde treba uplatniť vzorec (2.4 a 2.5), pričom symbol  $k$  označuje počet stromov s daným stupňom poškodenia a  $m$  celkový počet hodnotených stromov na TMP. Pre odhad priemernej hodnoty defoliácie  $\hat{y}$  na stromoch zo všetkých TMP je správny vzorec (2.7) a pre varianciu  $\hat{y}$  výraz (2.5), v ktorom sa namiesto  $k_{i;ha}$  dosadí  $y_{i;ha}$  a namiesto  $\hat{p}$  hodnota  $\hat{y}_{ppz}$ .

V ďalších kapitolách sa táto teória aplikuje na konkrétne údaje z výskumného objektu Orava, pričom kvôli porovnaniu sa všetky odhady robia dvojako: A – správnym spôsobom so zohľadnením nerovnakej veľkosti skusných plôch a B – nesprávnym spôsobom bez zohľadnenia tejto skutočnosti. Výsledky sú zhrnuté v tabuľkách a týkajú sa hektárového počtu stromov a zásoby porastov (tab. II), percentuálneho zastúpenia stromov v stupňoch poškodenia 0, 1, ... 4 (tab. III) a priemernej hodnoty straty asimilačných orgánov (%) v celom objekte (tab. IV). Osobitná pozornosť sa venuje stanoveniu zmeny monitorovaných veličín počas monitorovacieho cyklu (podľa 3.2 a 3.4), odvodením strednej chyby tejto diferencie (podľa 3.1, 3.5 a 3.6) a testovaniu štatistickej významnosti zistenej diferencie (podľa 3.3). Pritom sa celková (brutto) zmena analyzuje tak, že sa rozkladá na zložky, ktoré vznikli rastovým procesom (ako netto zmena), ťažbovou činnosťou a prirodzenou mortalitou. Všetky výsledky sú prezentované rovnakou formou – stav v roku 1991 a 1995 ako aj príslušné zmeny sú charakterizované priemernou hodnotou a jej strednou výberovou chybou. Z ich porovnania jednoznačne vyplýva, že medzi spôsobmi hodnotenia A a B vznikajú také systematické rozdiely, ktoré nemožno tolerovať.

Záverom sa zdôrazňuje, že v prípade, keď sa produkčný a zdravotný stav lesa monitoruje pomocou skusných plôch o variabilnej veľkosti a ich výber sa uskutočňuje podľa pravidelnej systematickej siete, treba pri zhodnocovaní získaných výsledkov najprv prepočítať všetky údaje zo skusných plôch na rovnakú výmeru 1 ha a pre odhady priemerných hodnôt monitorovaných veličín použiť v príspevku navrhnuté postupy a vzorce.

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