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General considerations on the use of allometric models for single tree biomass estimation

8th FIA Symposium of the USDA Forest Service,
October 16-19 Monterey CA

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General Questions

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- Which facts should be considered if one wants to compare or integrate process based models and empirical approaches for biomass estimation?
- What are the implications of allometry and are they considered adequately in empirical data analysis?
- Why might there be problems to prove the assumptions of process models with empirical data?

Approaches

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- Research in single tree biomass estimation can be divided into two major motivations:
  - ***Process models*** aim to explain the development and partitioning of single tree biomass based on physical, mechanical and/or hydraulical processes,
  - ***Empirical research*** has the goal to explain the variance in a given dataset by means of a number of easy to measure independent variables.

Empirical research

- Empirical research of the last decades delivered a large variety of biomass functions for the most important tree species.
  - mathematical model formulation is very diverse,
  - datasets are often small,
  - hence, the validity of these models is mostly restricted to the given site conditions.
- For biomass estimation on regional or national level one would need more general approaches.

Process models

- Beside empirical models we know theoretical process models based on fractal geometry

  (e.g. West, Brown, Enquist 1999, Valentine & Mäkelä 2005).

- Under certain assumptions of relations inside a self similar fractal-like tree structure, the theoretical scaling between diameter and mass is supposed to be:

$$D \propto M^{3/8} \Rightarrow M \propto D^{2.667}$$

Allometric Models

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- Mathematical description of scaling relations in organisms (Huxley 1824; Snell 1892)
  - Scaling exponent ***b*** is a measure for the relation of two relative growth rates,
  - ***a*** = integration constant.
- Allometry is defined as: The study of the relative growth of ***a part*** of an organism in relation to the growth of ***the whole***.

$$M = a \cdot D^b$$

$$\frac{\delta D}{D} = b \cdot \frac{\delta M}{M}$$

Example

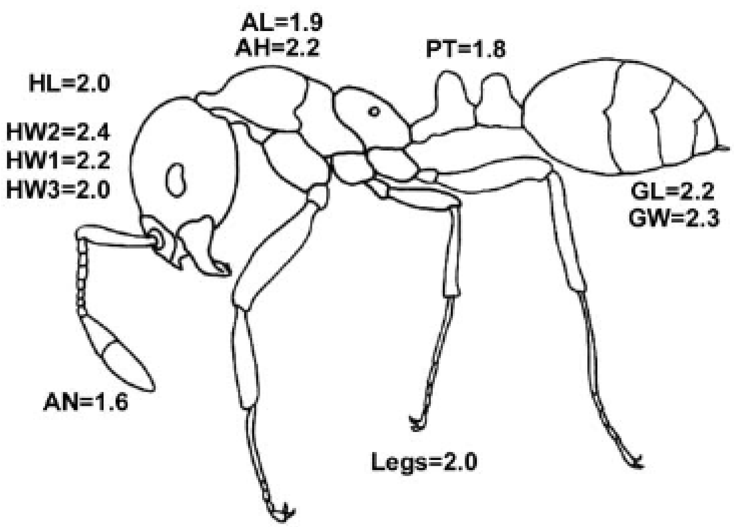

Figure 8. Side view of a fire ant worker of *Solenopsis invicta* summarizing the size increase of each body part for a doubling of total body length. A value of 2.0 indicates isometry, less than 2.0 negative allometry and greater than 2.0, positive allometry.

Tschinkel et al. 2003

Allometric Models

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- Log transformation of variables leads to linearity (homoscedasticity during regression analysis):

$$\ln M = \ln a + b * \ln D$$

- A linear trend of log transformed variables is often considered as argument to choose an allometric model (without rethinking about the theoretical background of allometry!).

Comparison of approaches

- Both approaches can be expressed in form of allometric equations, but:
  - In practice empirical data analysis leads to more or less different scaling factors than predictions of process models.

- Linking process based models and empirical data analysis might help to get more general approaches in form of hybrid models.

Differences

- To compare and/or integrate the different approaches, one has to think about the different motivations:
  - Process models aim to explain ontogenetic growth relations (inside one organism)!
  - Empirical data analysis is always based on destructive sampling! That means: data are collected in comparative observational studies (*chronosequences*) where trees of different dimensions are measured in one point in time.

Some simple geometry

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- Imagine trees could be approximated by the simple geometric form of a cone:
  - Under the assumption that height (H) scales with diameter D:

$$H = k\,D^{b'} \quad with\; b'\; being\; 2/3$$

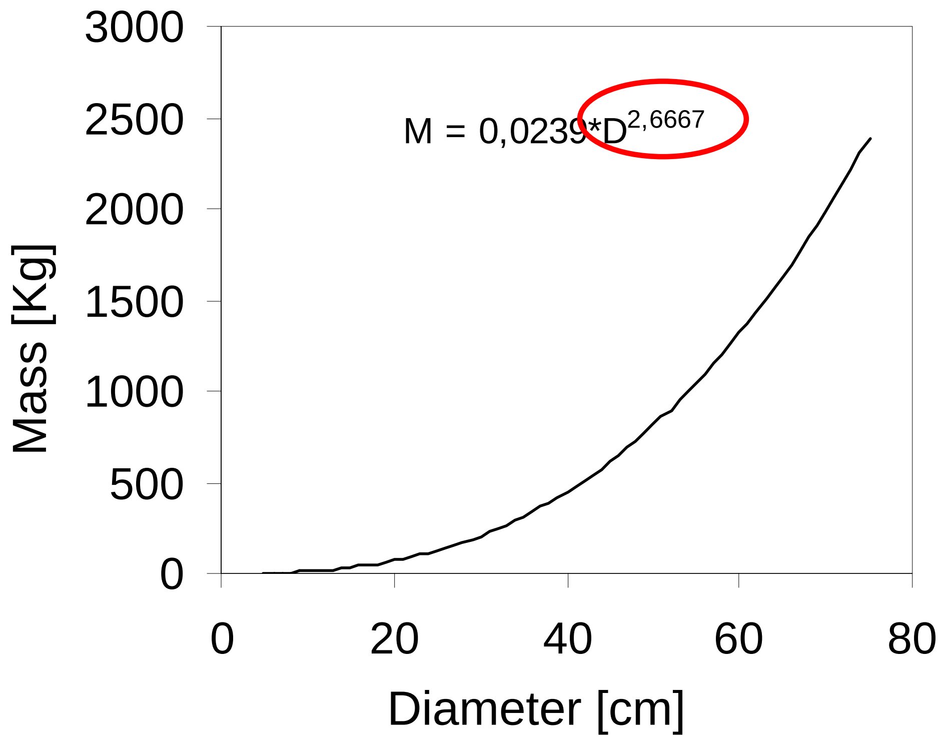
  - and volume

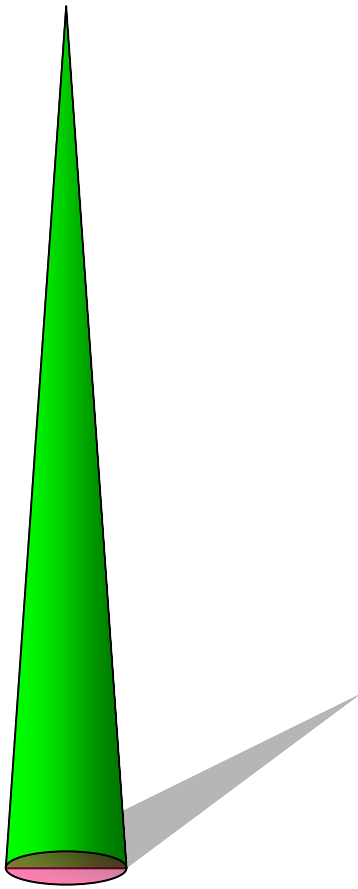
$$V_{cone} = \frac{\pi}{12} D^2\, h$$

Some simple geometry

- The mass of a cone can be calculated as:

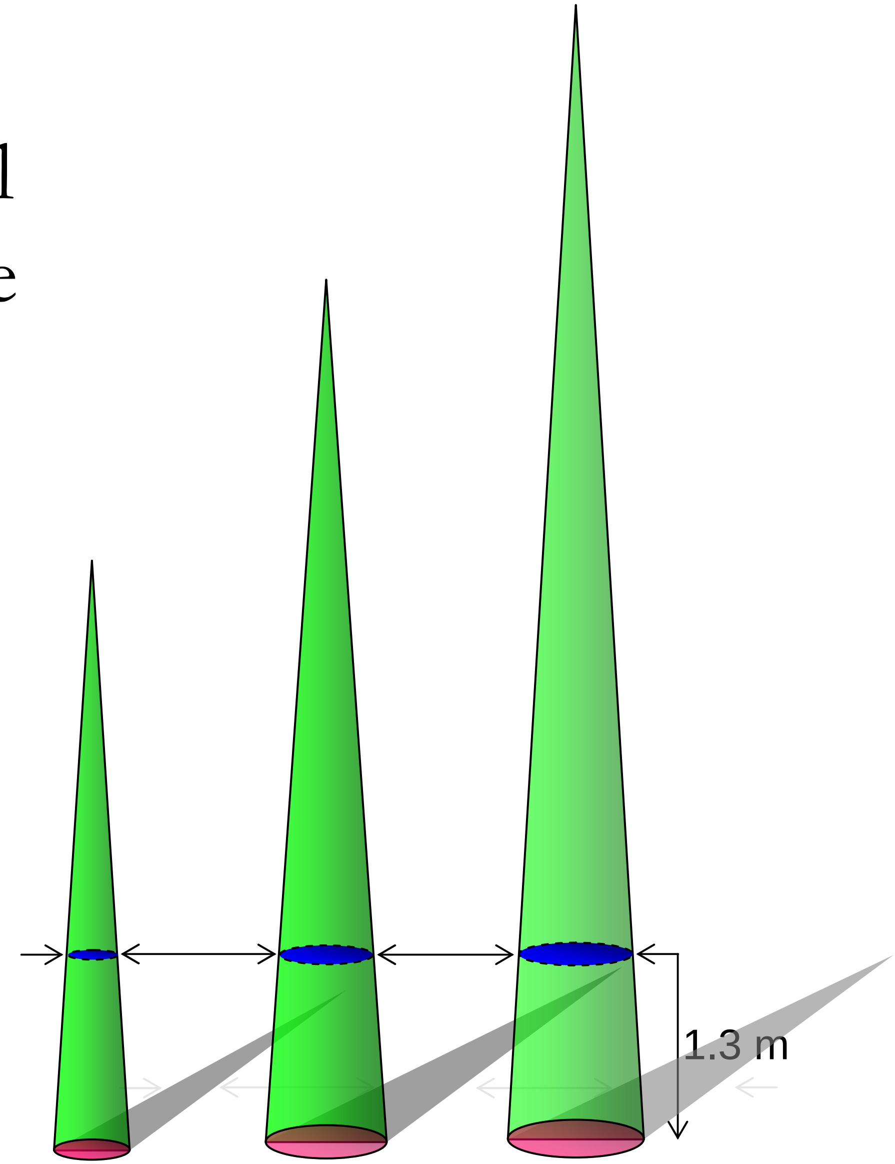
$$M_{cone} = \frac{\pi}{12} k \rho \cdot 0.1 D^{2+b'}$$





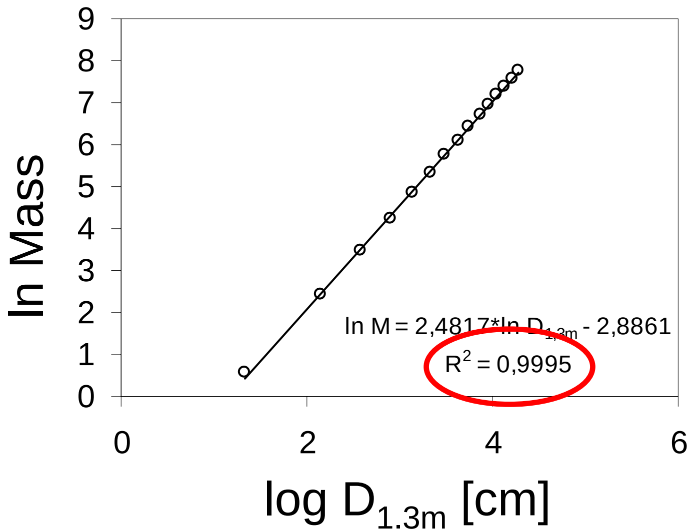
Some simple geometry

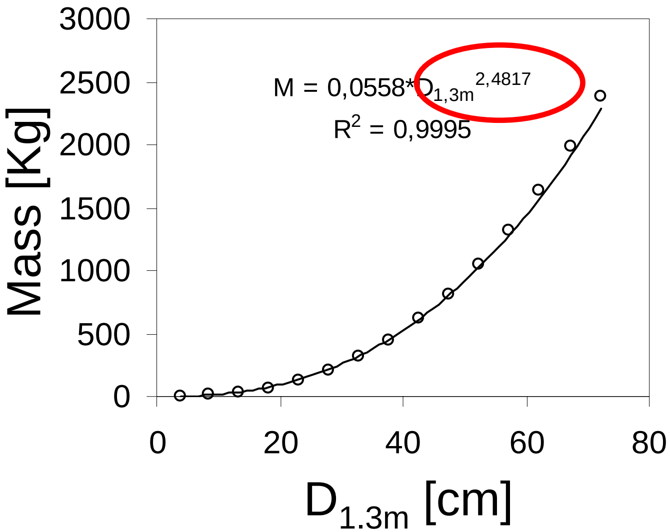
- Why can that theoretical assumption not easily be proven with empirical tree data?
  - Because empirical data give no information about relative growth rates of a functional diameter?!



Implications

- **In the sense of allometry the dbh is not a functional measure of a tree**!
- The deviation from linearity is hardly detectible in the logarithmic data, because of the balancing effect of the transformation
- For this example diameter in 1.3 m was used:



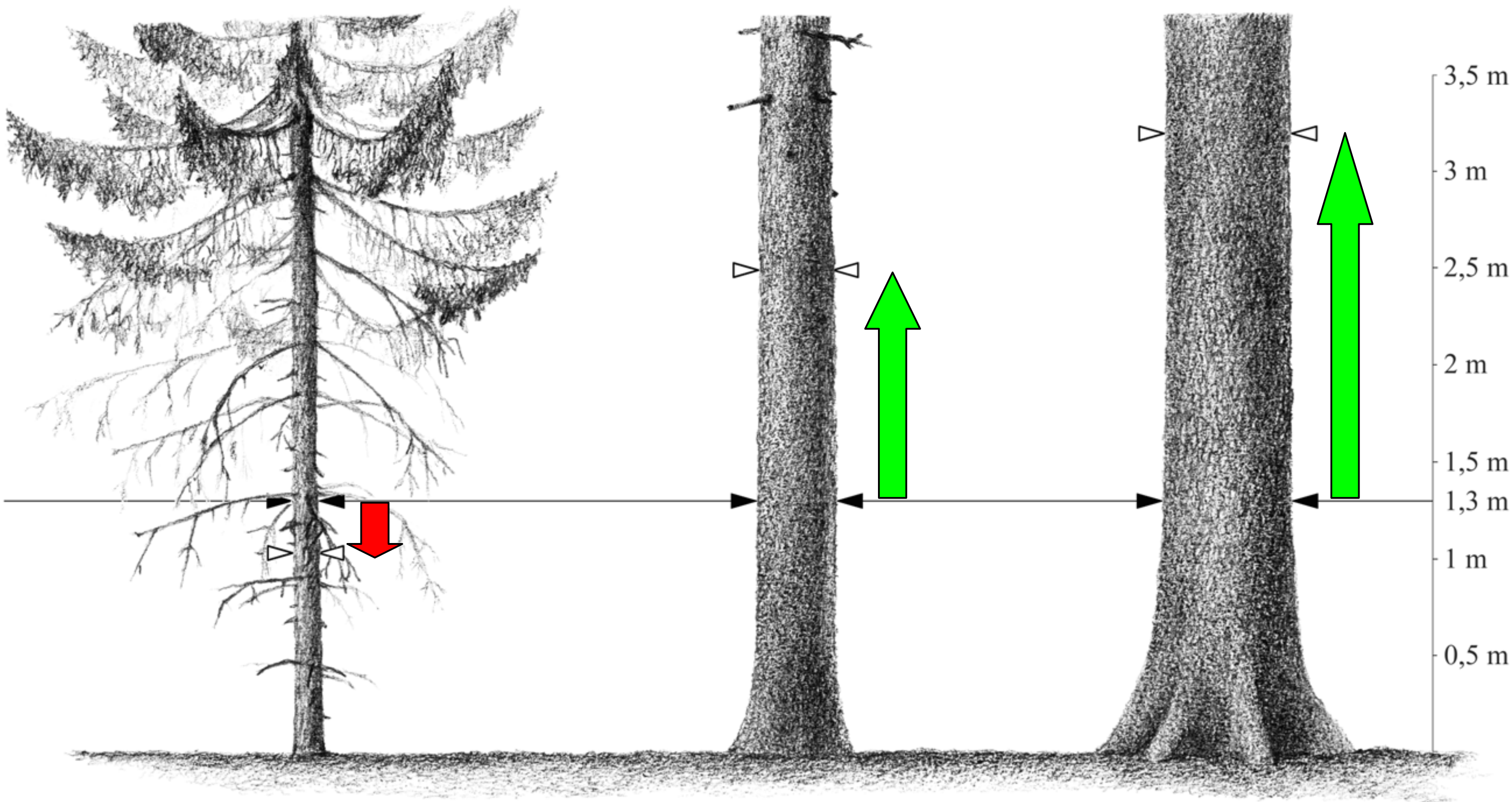


Example

- In a dataset of 310 Norway spruce trees from different sites in central Europe diameter in relative height ($d_{0.1}$) was modelled from dbh with an appropriate taper model (Pain and Boyer, 1996).

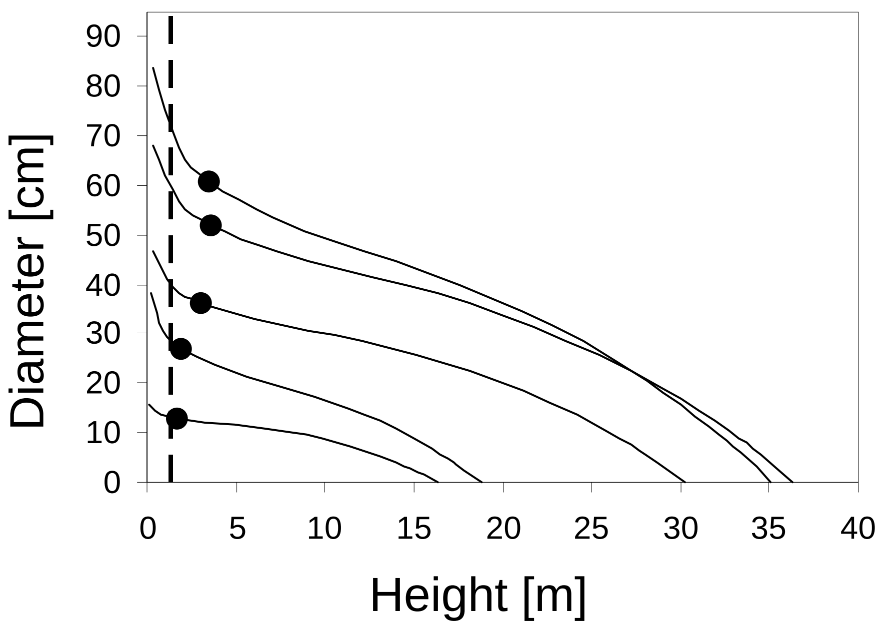
- Differences between dbh and $d_{0.1}$ range from only some millimeters for small trees up to 13 cm for big individuals.

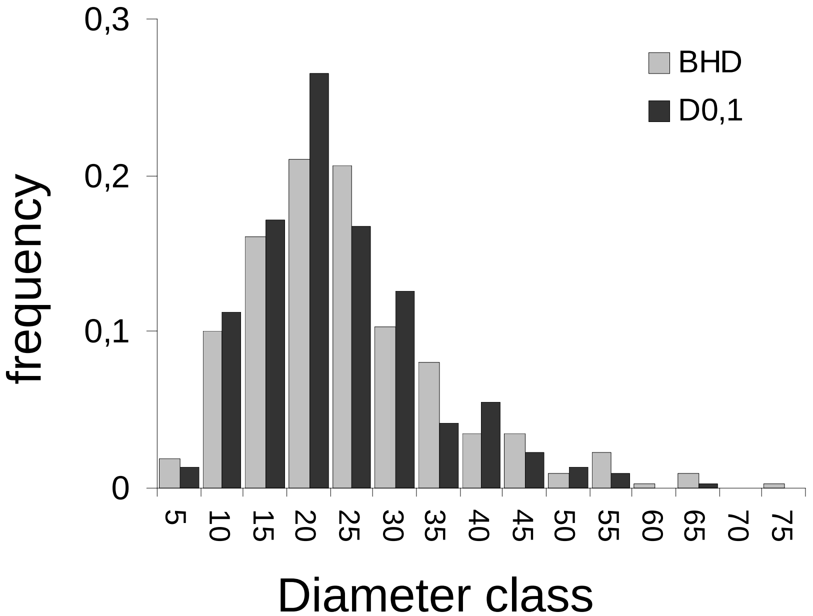
Example



Example

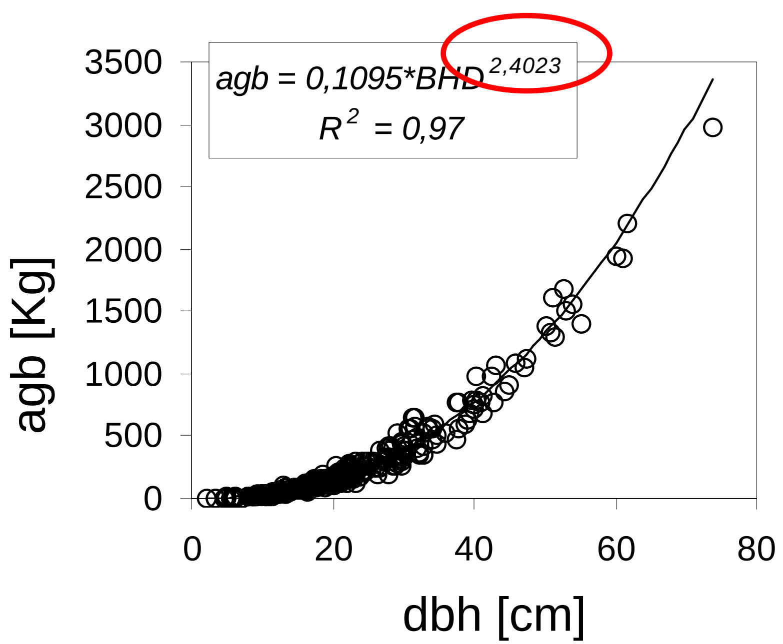
- The relative diameters $d_{0.1}$ are much less influenced by variations of taper form and allow a better insight into relative growth rates.

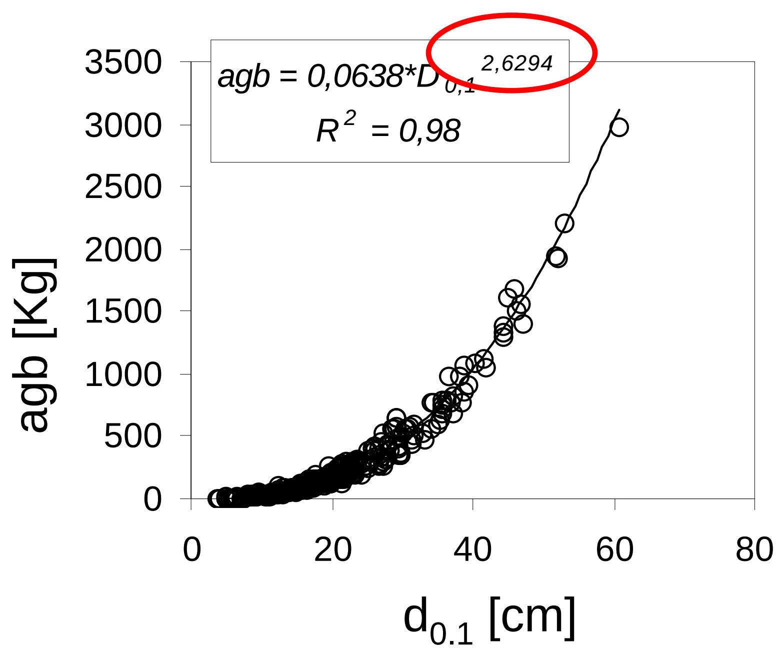




Result

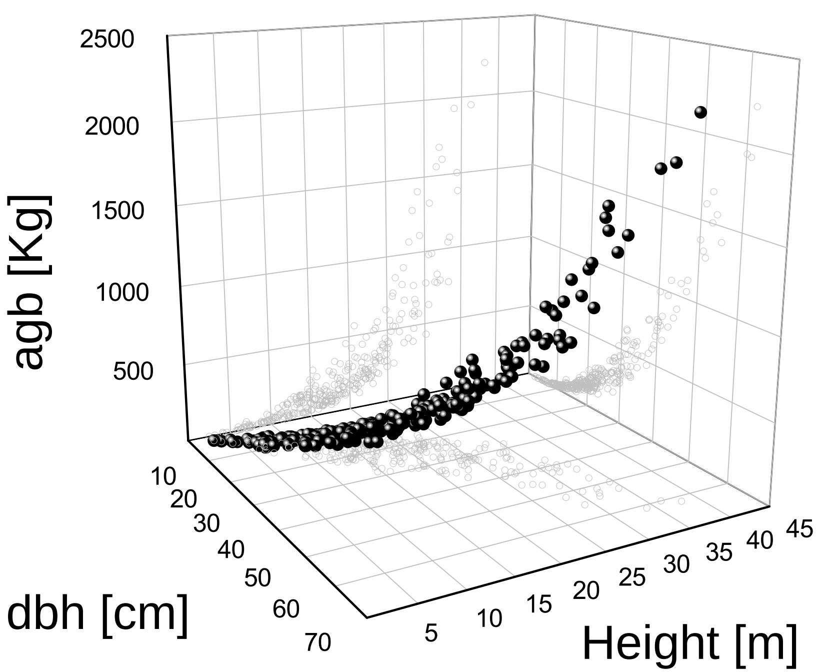
- Model performance is slighly enhanced when using $d_{0.1}$ as independent variable,
- The scaling factor is closer to the expected value of 2.667.

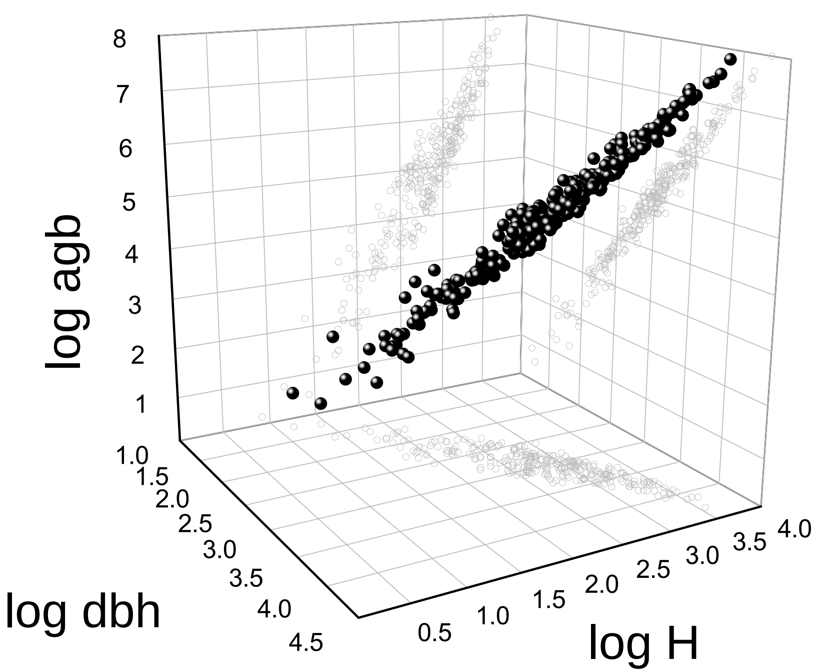




Additional aspects

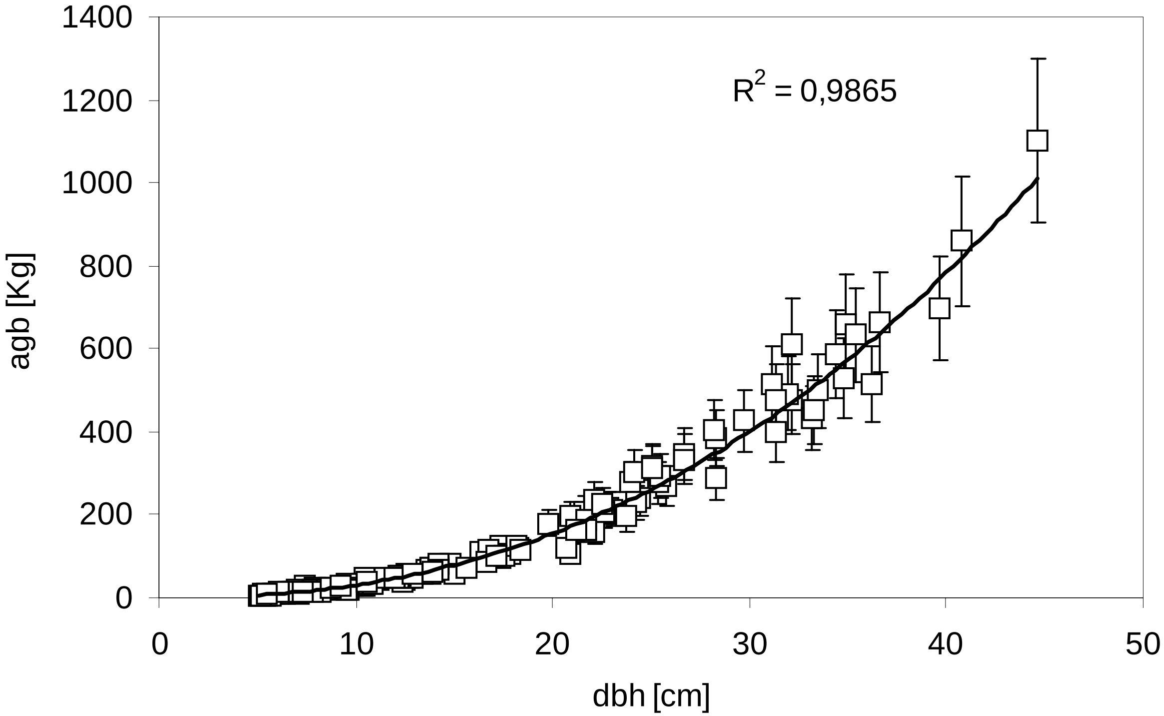
- For small study sites, tree height does not enhance model performance (because of the high correlation with dbh).
- This is clearly different when various data sets are combined.





Additional aspects

- What do confidence intervalls of biomass equations are telling us, when we remember that the „observed“ values are based on sampling on single tree level?



- Thank you!

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