# RING
# Gradient Delay Estimation in Radial Imaging

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NOTE

Magnetic Resonance in Medicine

**Simple auto-calibrated gradient delay estimation from few spokes using Radial Intersections (RING)**

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UNIVERSITÄTSMEDIZIN GÖTTINGEN UMG
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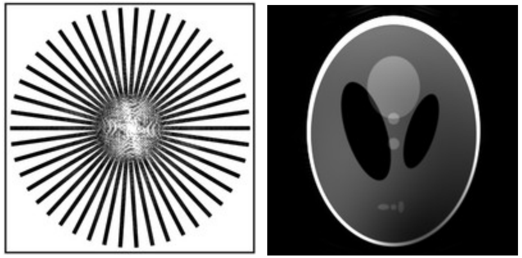
GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

# Motivation

CARTESIAN

RADIAL



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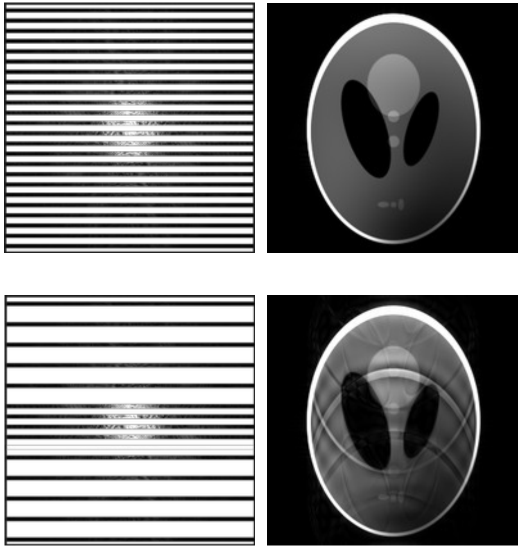
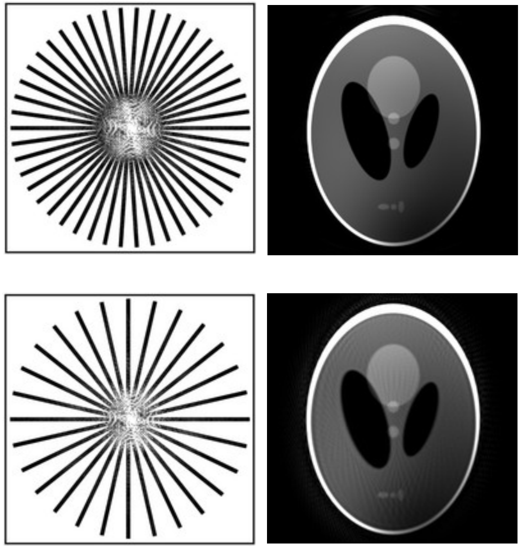
CARTESIAN



RADIAL

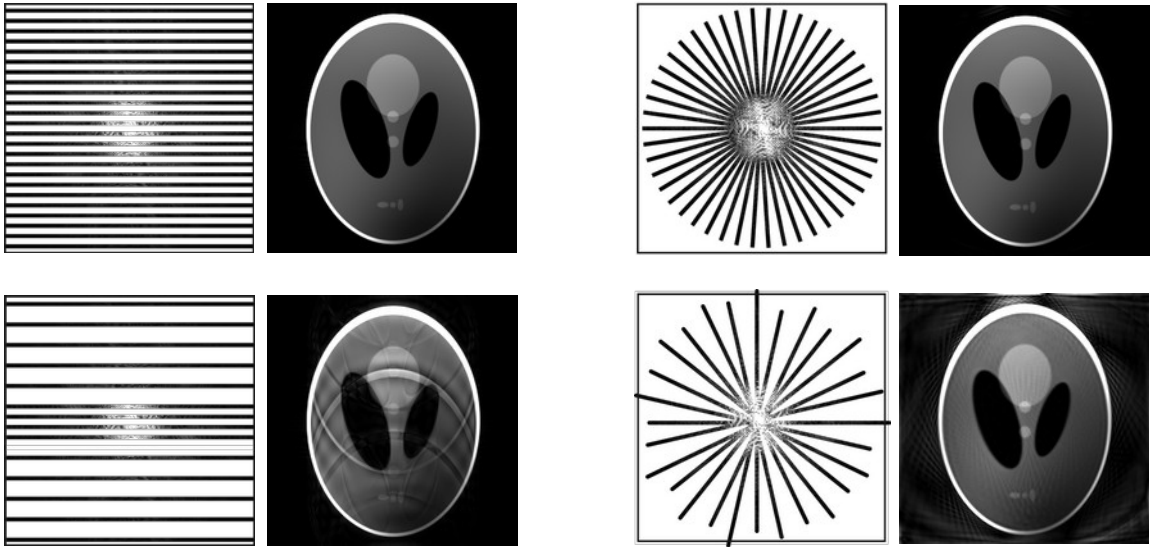


- + less undersampling artifacts
- + motion robustness

# Motivation

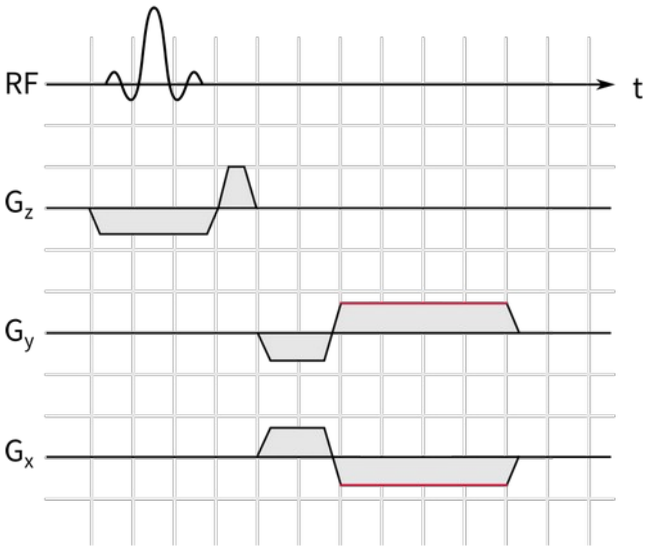
CARTESIAN

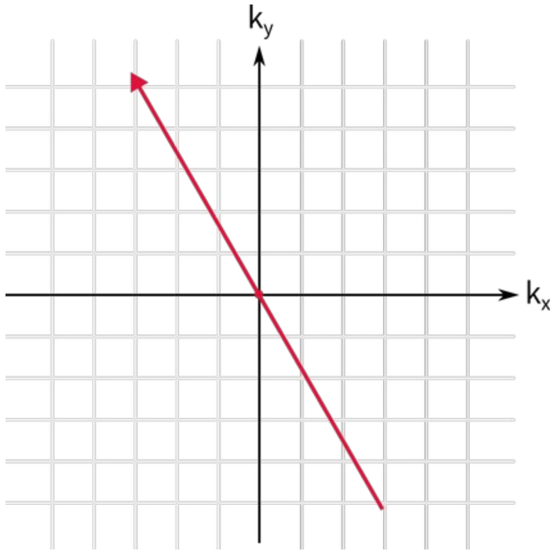
RADIAL



- prone to system imperfections

# k-Space Shift



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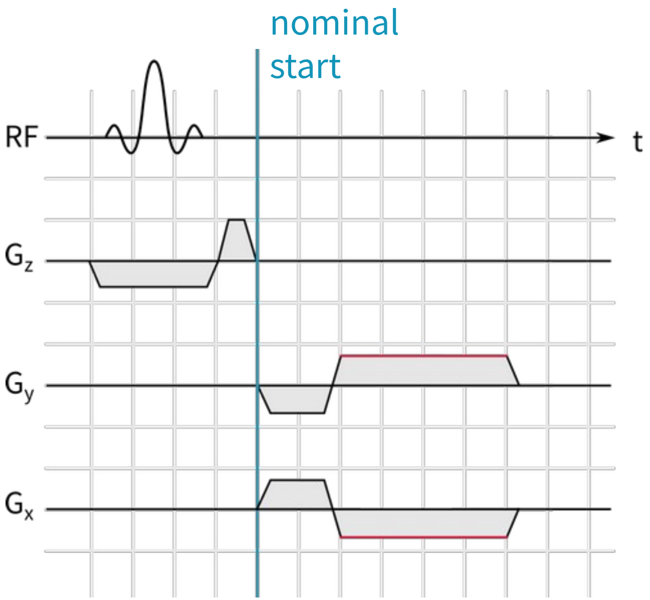


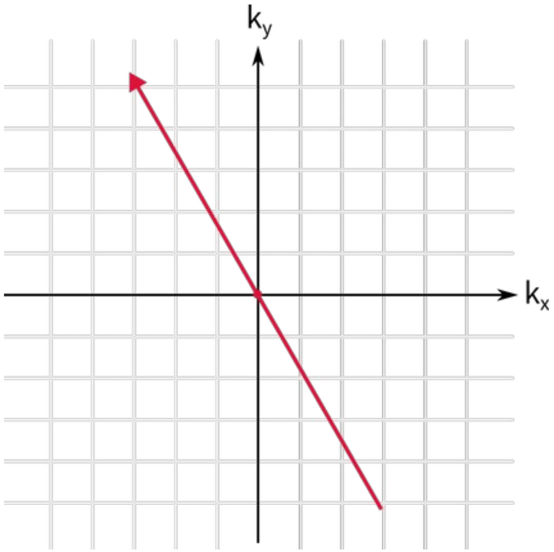

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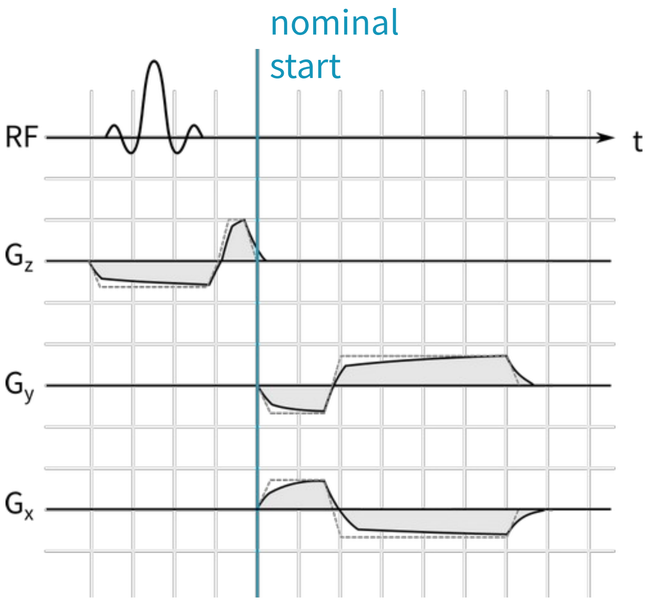


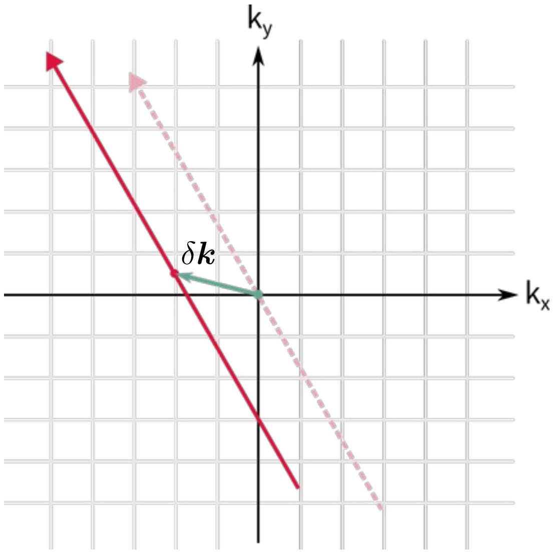

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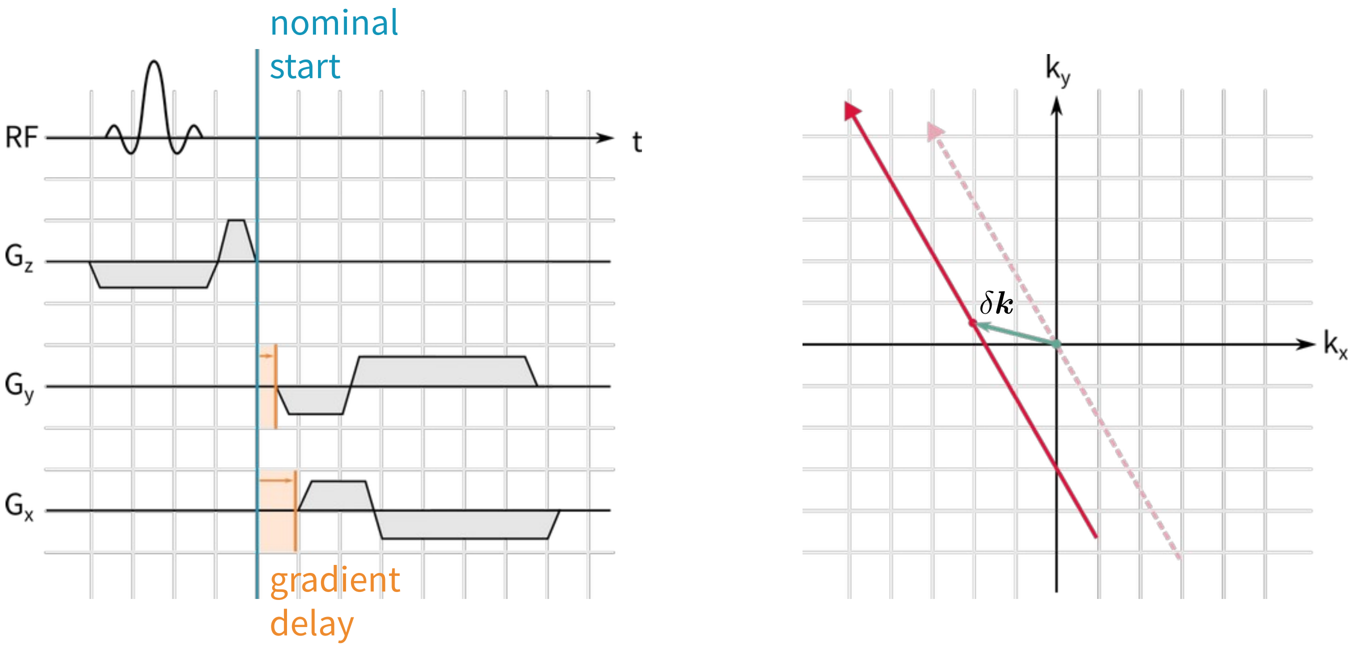

# Ellipse Model

- k-space shift

$$\delta \boldsymbol{k} = \boldsymbol{S} \hat{\boldsymbol{n}}_\theta$$

- delay matrix[1]

$$\boldsymbol{S} = \begin{pmatrix} S_x & S_{xy} \\ S_{xy} & S_y \end{pmatrix}$$

- projection direction

$$\hat{\boldsymbol{n}}_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

[1] Moussavi et al., Magn. Reson. Med. (2013)

# Ellipse Model

- k-space shift

$$\delta \boldsymbol{k} = \boldsymbol{S}\hat{\boldsymbol{n}}_\theta$$

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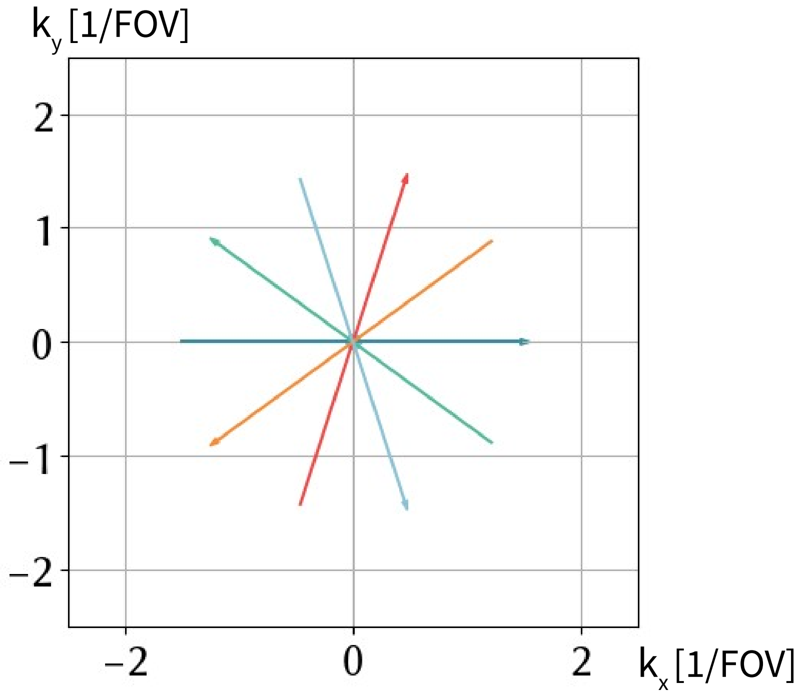


[1] Moussavi et al., Magn. Reson. Med. (2013)

# Ellipse Model

- k-space shift

$$\delta \boldsymbol{k} = \boldsymbol{S}\hat{\boldsymbol{n}}_\theta$$

- delay matrix[1]

$$\boldsymbol{S} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

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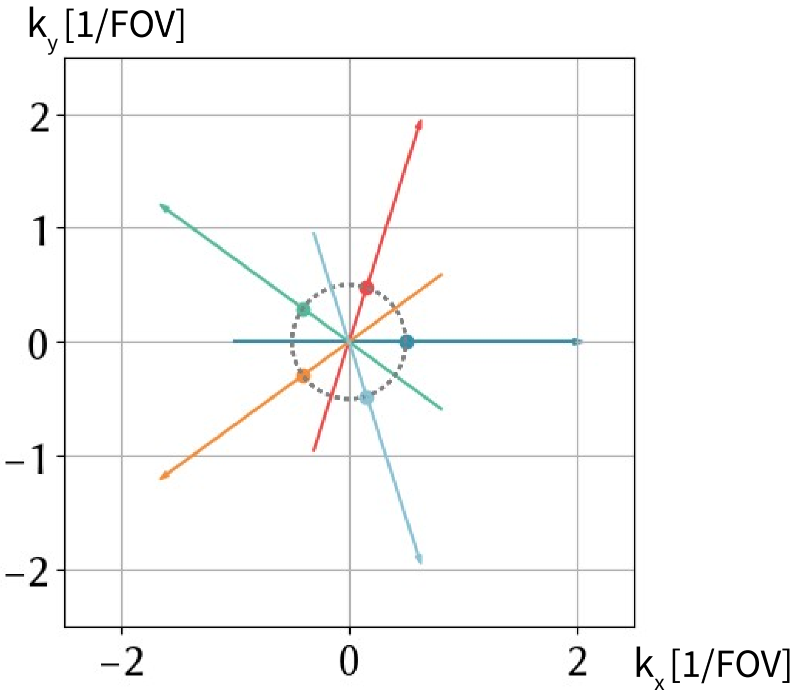


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$$\delta \boldsymbol{k} = \boldsymbol{S}\hat{\boldsymbol{n}}_\theta$$

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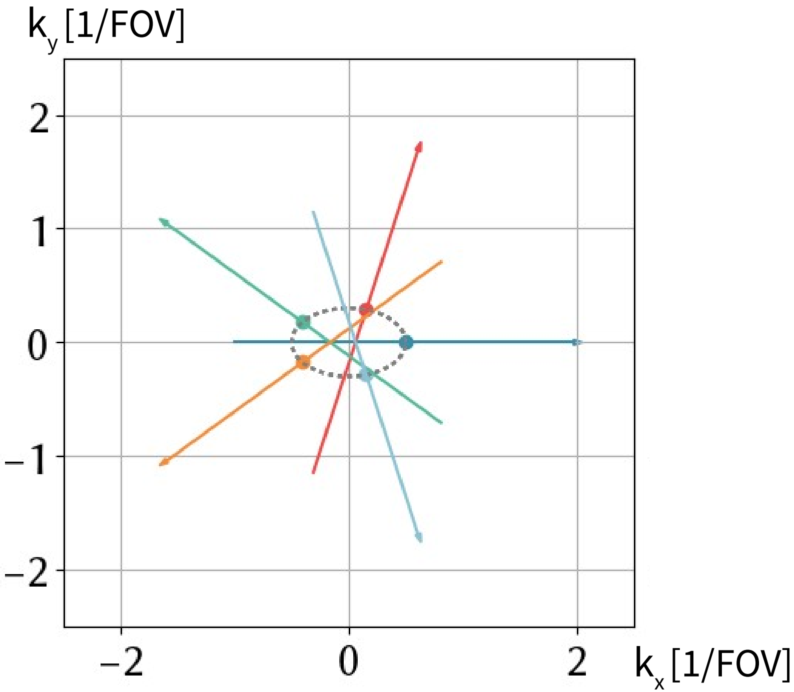


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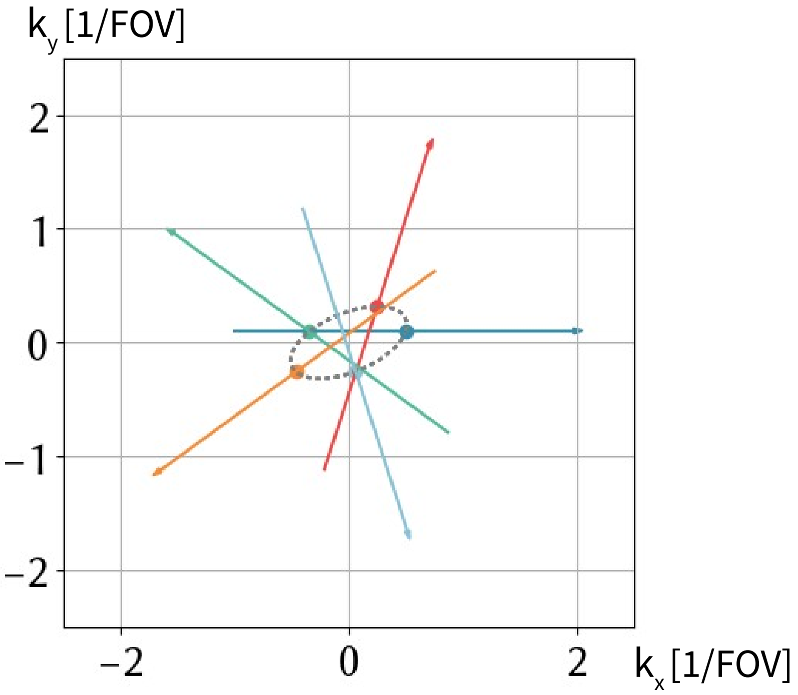


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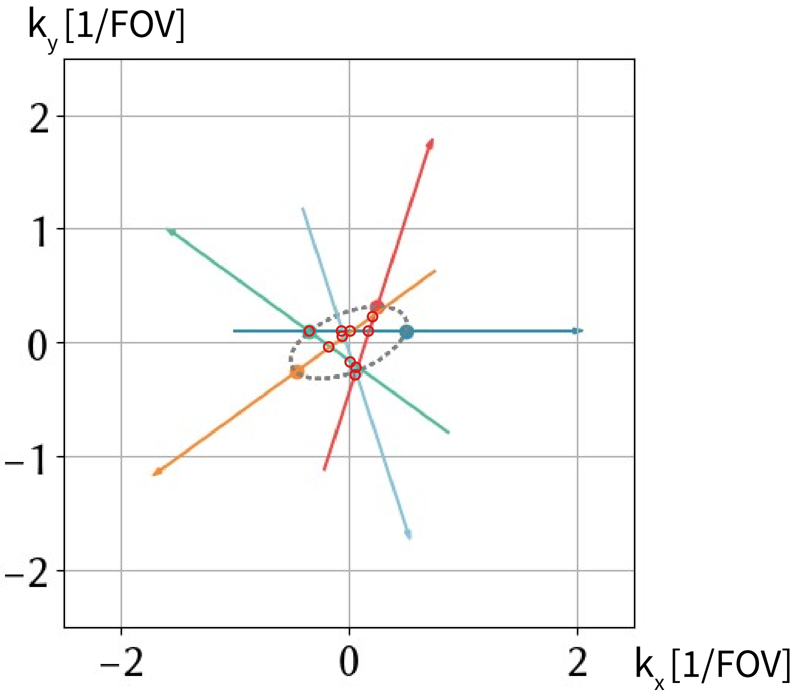


intersection points carry information about $\boldsymbol{S}$

[1] Moussavi et al., Magn. Reson. Med. (2013)

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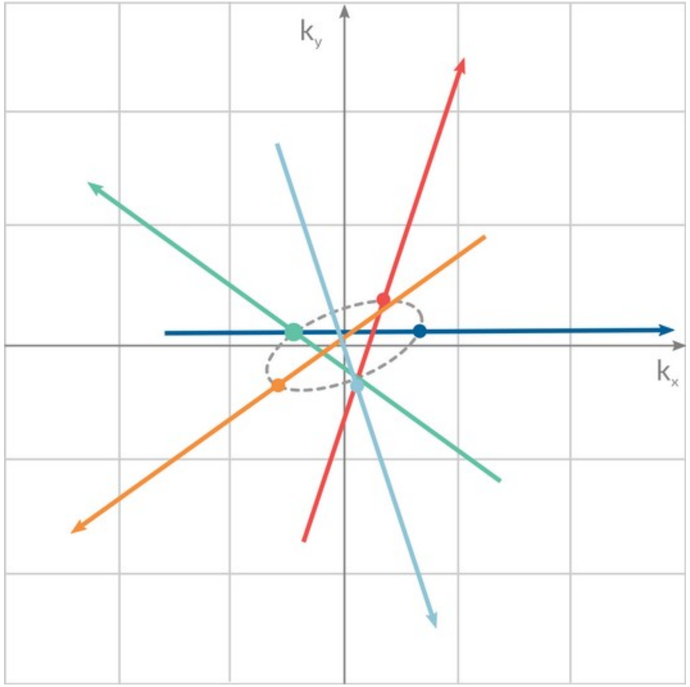
STRATEGY

1) determine intersection points
2) use least-squares fit to obtain $\boldsymbol{S}$
3) use $\boldsymbol{S}$ to determine actual trajectory $P(\boldsymbol{S})$

IMAGE RECONSTRUCTION

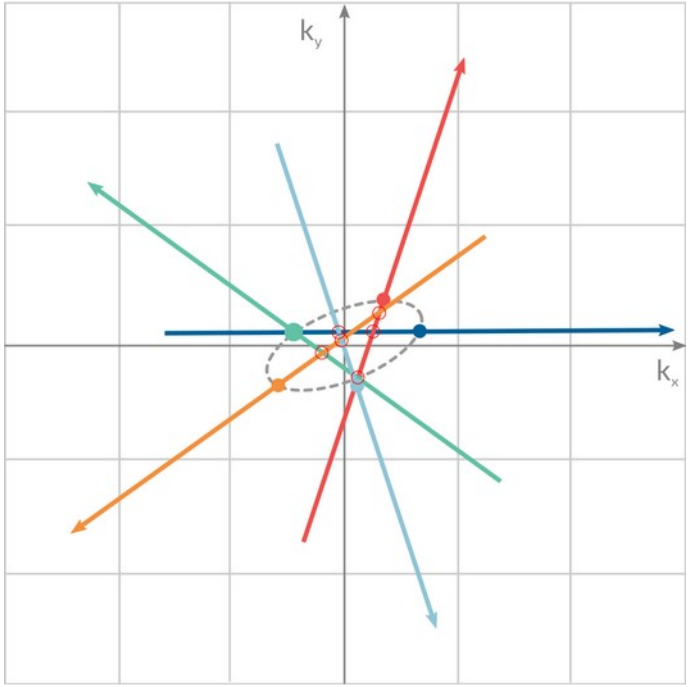
$$\underset{\boldsymbol{x}}{\operatorname{argmin}} ||P(\boldsymbol{S})\mathcal{F}C\boldsymbol{x} - \boldsymbol{y}||^2 + R(\boldsymbol{x})$$

# Determine the Shift



# Determine the Shift

- consider the intersection points



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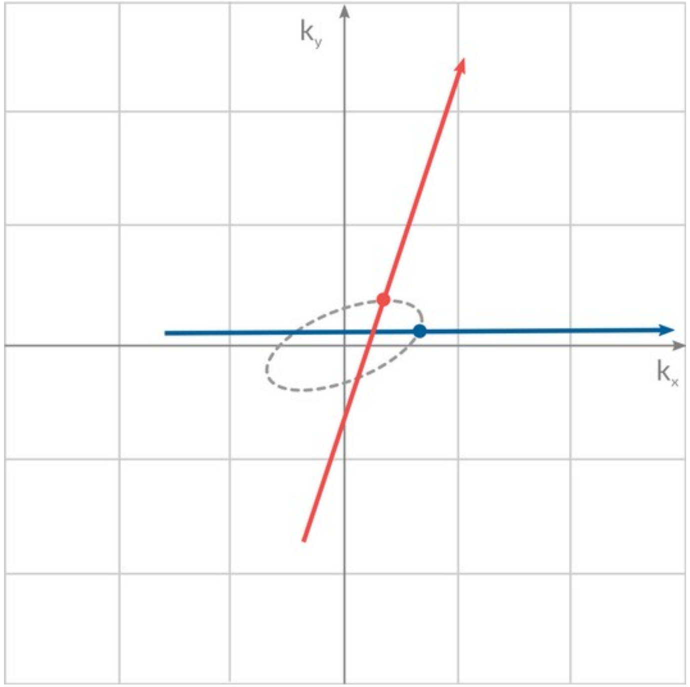

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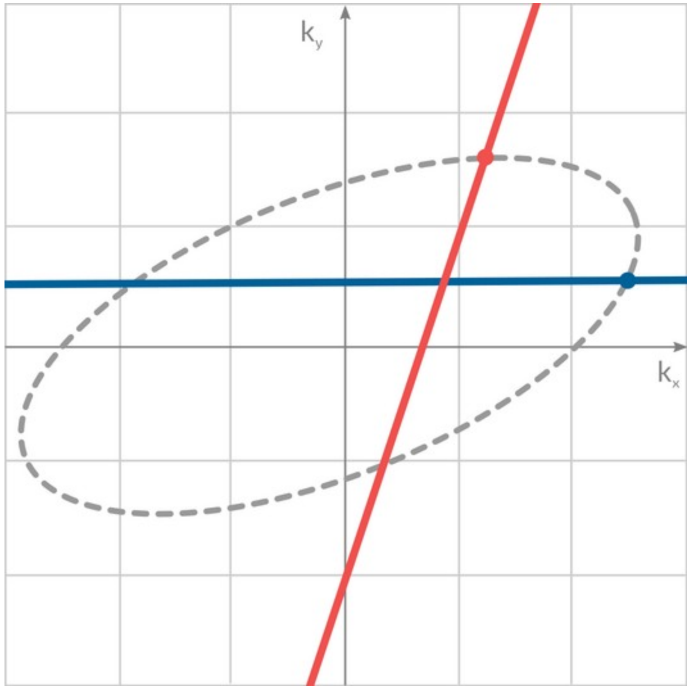
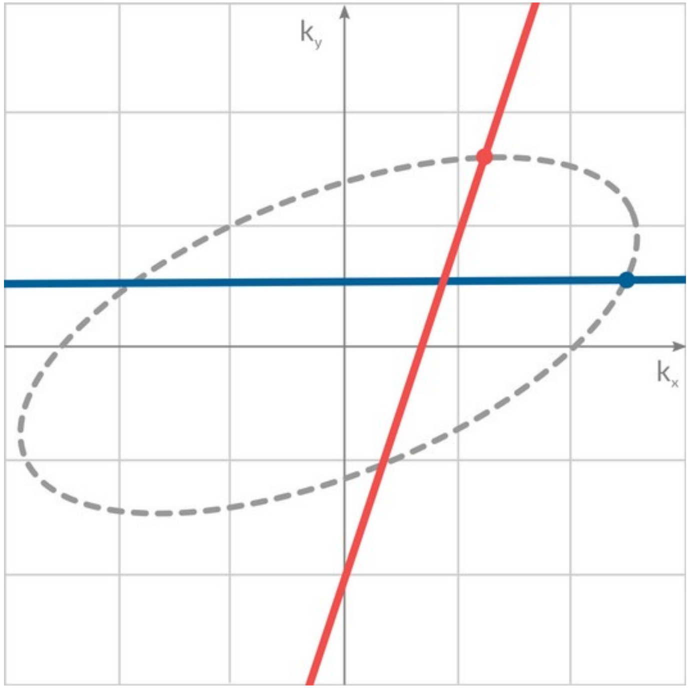


# Determine the Shift

- consider the intersection points
- spoke parametrization

$$\boldsymbol{r}_{\theta_i} = \boldsymbol{S}\hat{\boldsymbol{n}}_{\theta_i} + a_{\theta_i}\hat{\boldsymbol{n}}_{\theta_i}$$



# Determine the Shift

- consider the intersection points
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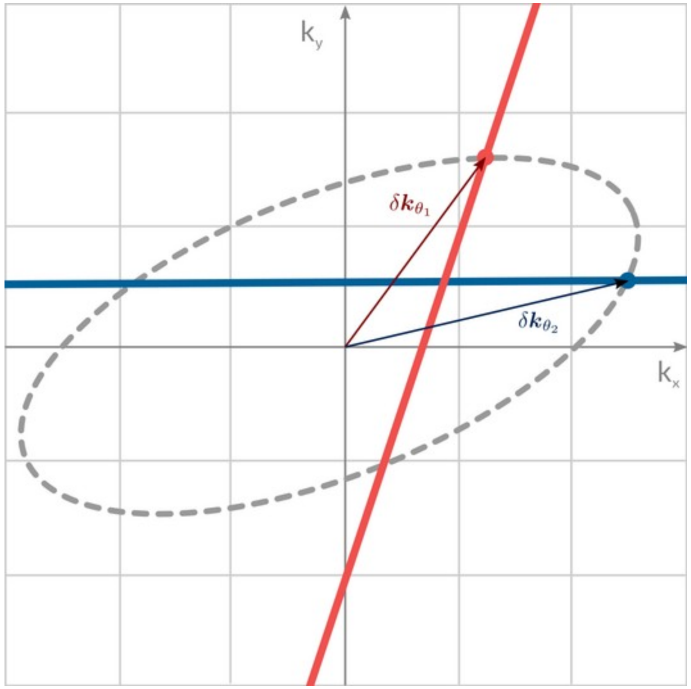
$$\boldsymbol{r}_{\theta_i} = \boldsymbol{S}\hat{\boldsymbol{n}}_{\theta_i} + a_{\theta_i}\hat{\boldsymbol{n}}_{\theta_i}$$

shift $\delta \boldsymbol{k}_{\theta_i}$



# Determine the Shift

- consider the intersection points
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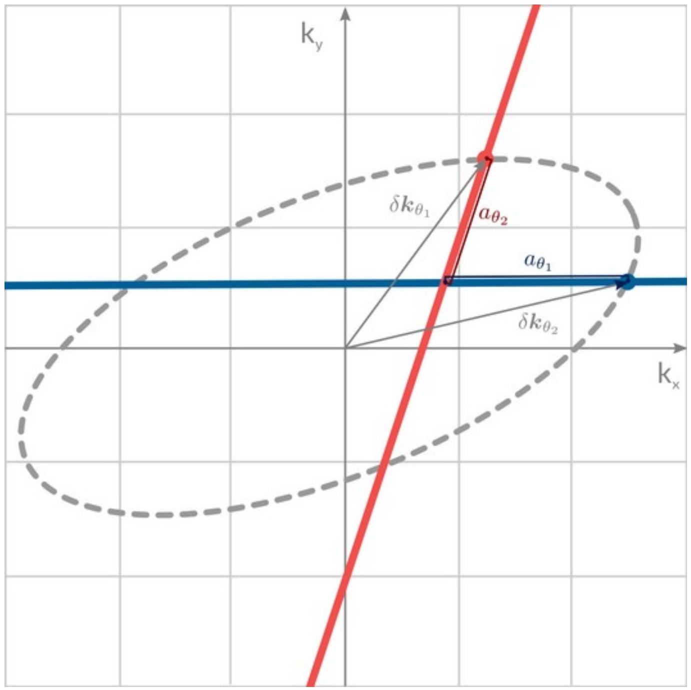
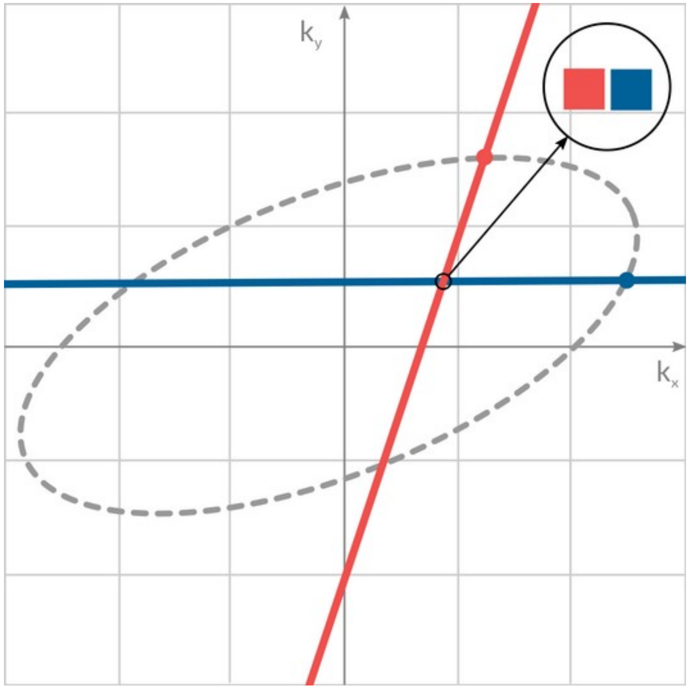
$$\boldsymbol{r}_{\theta_i} = \underbrace{\boldsymbol{S}\hat{\boldsymbol{n}}_{\theta_i}}_{\text{shift } \delta\boldsymbol{k}_{\theta_i}} + \underbrace{a_{\theta_i}\hat{\boldsymbol{n}}_{\theta_i}}_{\text{offset from spoke-center}}$$



# Determine the Shift

- consider the intersection points
- spoke parametrization
  $\boldsymbol{r}_{\theta_i} = \boldsymbol{S}\hat{\boldsymbol{n}}_{\theta_i} + a_{\theta_i}\hat{\boldsymbol{n}}_{\theta_i}$
- determine $a_{\theta_i}$ via pixel-by-pixel comparison



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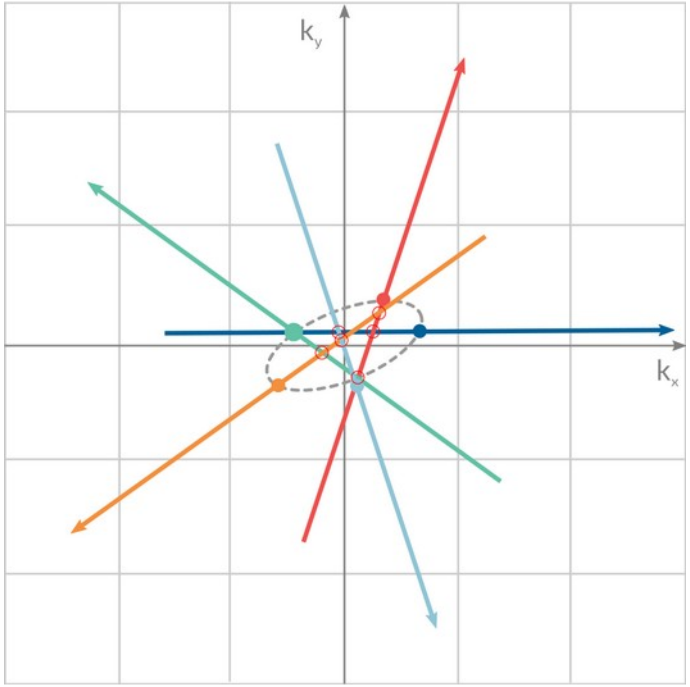
  $\boldsymbol{r}_{\theta_i} = \boldsymbol{S}\hat{\boldsymbol{n}}_{\theta_i} + a_{\theta_i}\hat{\boldsymbol{n}}_{\theta_i}$
- determine $a_{\theta_i}$ via pixel-by-pixel comparison
- repeat for all intersection points
  $\boldsymbol{r}_{\theta_i}(\boldsymbol{S}a_{\theta_i}) \stackrel{!}{=} \boldsymbol{r}_{\theta_j}(\boldsymbol{S}a_{\theta_j}) \quad \forall i, j$



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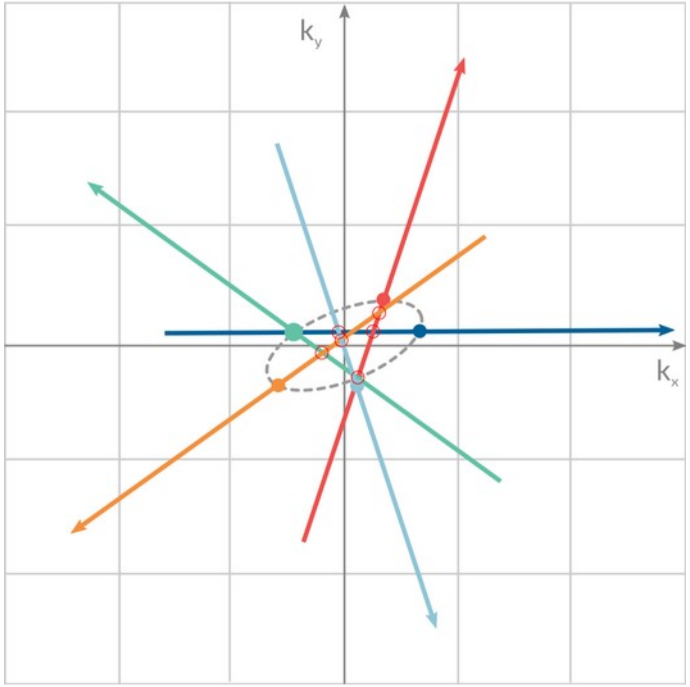
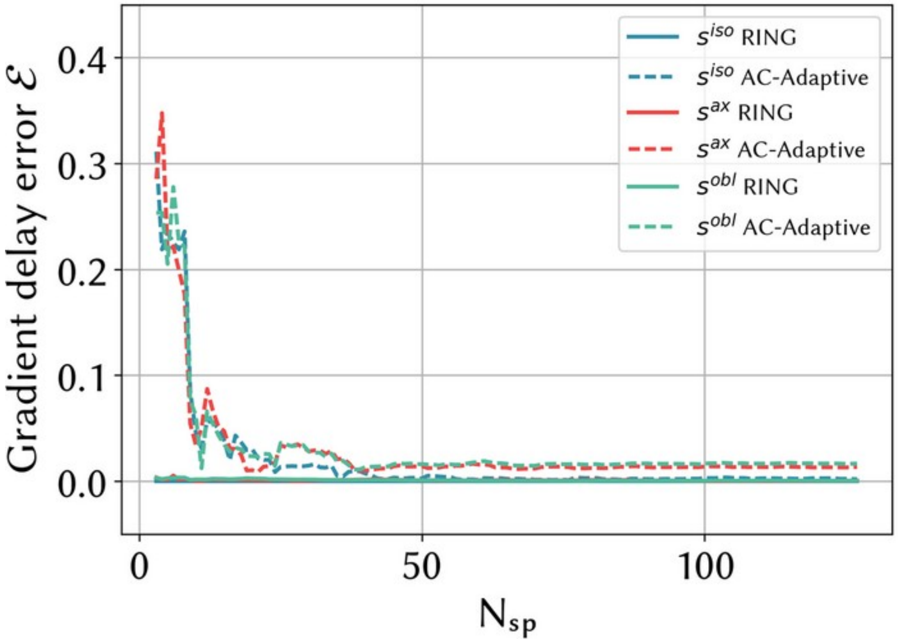
- repeat for all intersection points
  $\boldsymbol{r}_{\theta_i}(\boldsymbol{S}a_{\theta_i}) \stackrel{!}{=} \boldsymbol{r}_{\theta_j}(\boldsymbol{S}a_{\theta_j}) \quad \forall i, j$
- determine $\boldsymbol{S}$ by least-squares fit



# Numerical Simulation

- Shepp-Logan phantom
- AC-Adaptive method as state-of-the art reference[1]

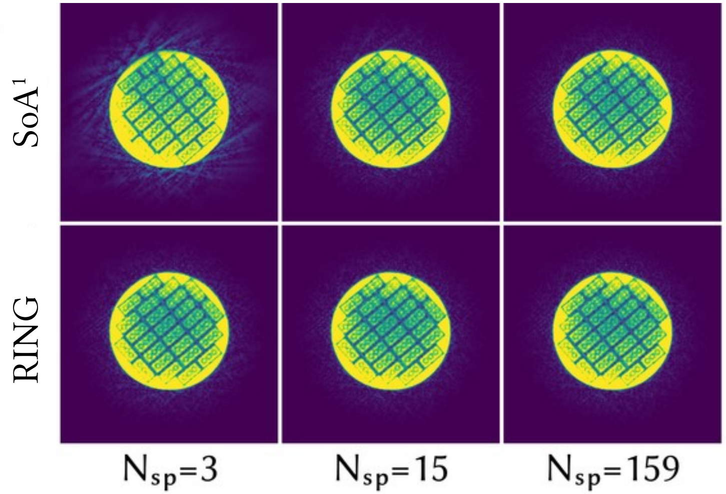


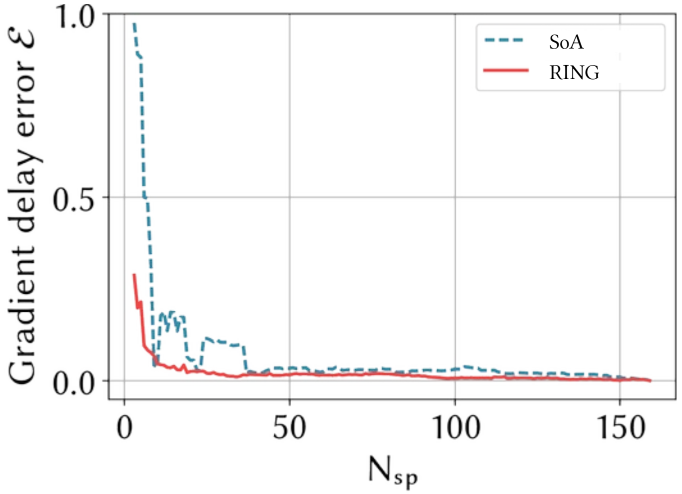

(number of spokes used for gradient delay estimation)

[1] Block and Uecker, ISMRM 2011

# Phantom Experiment

- Brick phantom
- $N_{sp}$ spokes used for gradient delay estimation




[1] Block and Uecker, ISMRM 2011

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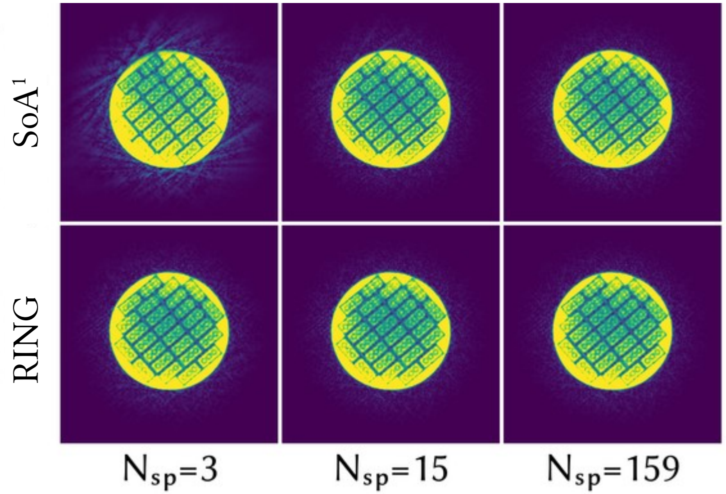
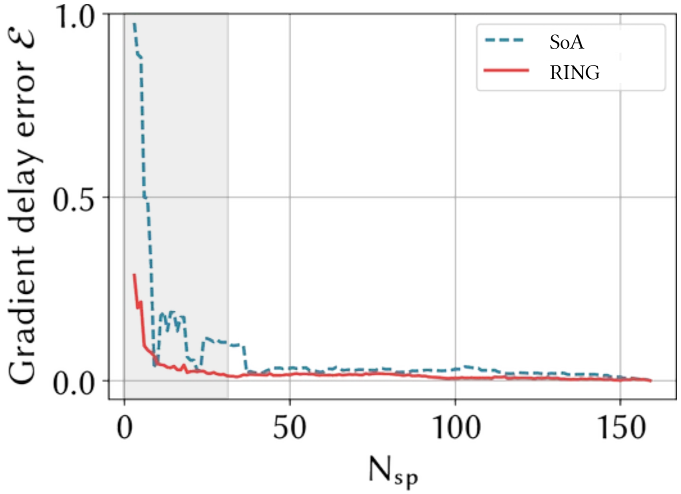


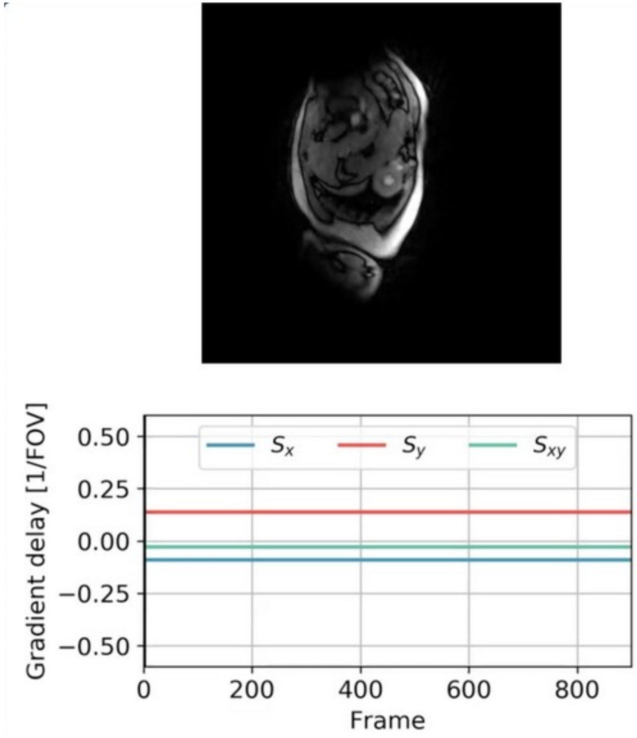




[1] Block and Uecker, ISMRM 2011

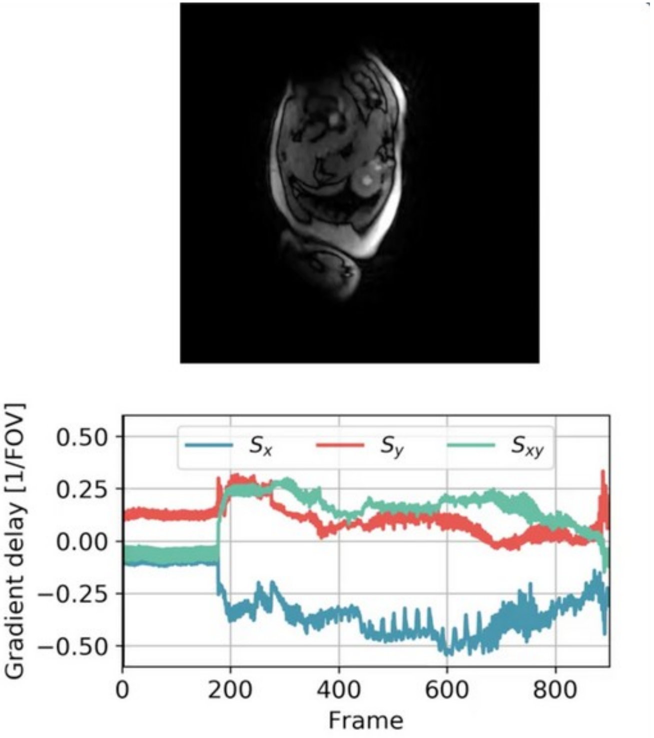
# Interactive Real-Time Experiment

21 spokes per frame[1]

**1st frame only**
gradient delay estimation




**frame-by-frame**
gradient delay estimation

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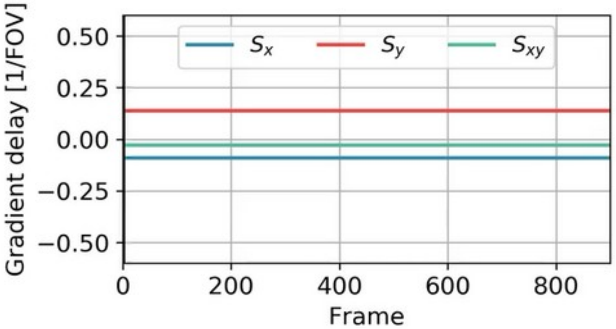


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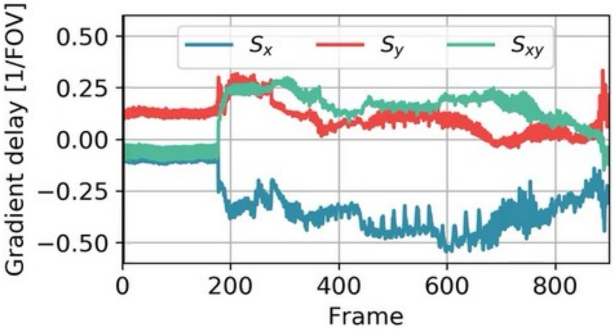



[1] Rosenzweig et al., ISMRM 2019

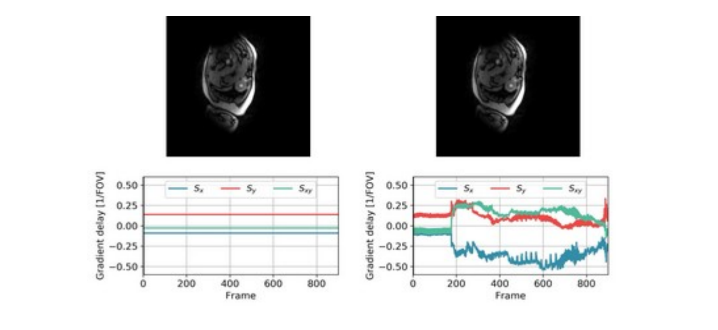
# Wrap Up

SUMMARY
- simple & intuitive
- very few spokes required

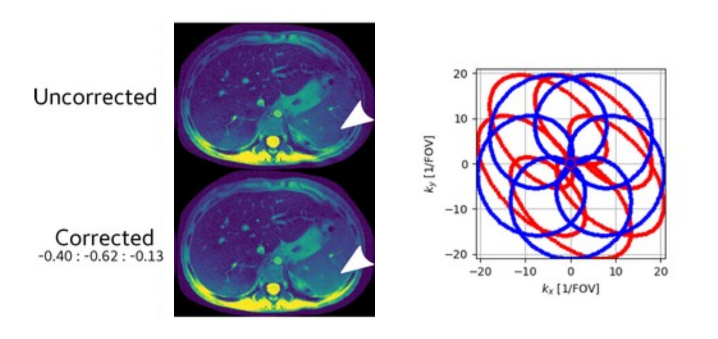
➔ interactive real-time MRI

LIMITATION
- offline study

OUTLOOK
- online integration
- extension to other trajectories[1]




RING applied to Rosette-trajectories[1]




[1] Roeloffs et al., ISMRM 2020