

nag_2d_scatter_interpolant (e01sac)

1. Purpose

nag_2d_scatter_interpolant (e01sac) generates a two-dimensional surface interpolating a set of scattered data points, using either the method of Renka and Cline or a modification of Shepard's method.

2. Specification

```
#include <nag.h>
#include <nage01.h>

void nag_2d_scatter_interpolant(Nag_2d_Scatter_Method method, Integer m,
                                double x[], double y[], double f[], Nag_Scatter_Struct *comm,
                                Nag_E01_Opt *optional, NagError *fail)
```

3. Description

This function constructs an interpolating surface $F(x, y)$ through a set of m scattered data points (x_r, y_r, f_r) , for $r = 1, 2, \dots, m$, using either a method due to Renka and Cline or a modification of Shepard's method. In the (x, y) plane, the data points must be distinct. The constructed surface is continuous and has continuous first derivatives.

If **method** = **Nag_RC**, the method due to Renka and Cline is used. This involves firstly creating a triangulation with all the (x, y) data points as nodes, the triangulation being as nearly equiangular as possible (see Cline and Renka (1984)). Then gradients in the x - and y -directions are estimated at node r , for $r = 1, 2, \dots, m$, as the partial derivatives of a quadratic function of x and y which interpolates the data value f_r , and which fits the data values at nearby nodes (those within a certain distance chosen by the algorithm) in a weighted least-squares sense. The weights are chosen such that closer nodes have more influence than more distant nodes on derivative estimates at node r . The computed partial derivatives, with the f_r values, at the three nodes of each triangle define a piecewise polynomial surface of a certain form which is the interpolant on that triangle. See Renka and Cline (1984) for more detailed information on the algorithm, a development of that by Lawson (1977). The code is derived from Renka (1984).

The interpolant $F(x, y)$ can subsequently be evaluated at n points (x_k, y_k) , for $k = 1, 2, \dots, n$, inside or outside the domain of the data by a call to **nag_2d_scatter_eval** (e01sbc). Points outside the domain are evaluated by extrapolation.

If **method** = **Nag_Shep**, then a modification of Shepard's method is used. The basic Shepard method, described in Shepard (1968), interpolates the input data with the weighted mean

$$F(x, y) = \frac{\sum_{r=1}^m w_r(x, y) f_r}{\sum_{r=1}^m w_r(x, y)},$$

$$\text{where } w_r(x, y) = \frac{1}{d_r^2} \text{ and } d_r^2 = (x - x_r)^2 + (y - y_r)^2.$$

The basic method is global in that the interpolated value at any point depends on all the data, but this routine uses a modification due to Franke and Nielson (1980), whereby the method becomes local by adjusting each $w_r(x, y)$ to be zero outside a circle with centre (x_r, y_r) and some radius R_w . Also, to improve the performance of the basic method, each f_r above is replaced by a function $f_r(x, y)$, which is a quadratic fitted by weighted least-squares to data local to (x_r, y_r) and forced to interpolate (x_r, y_r, f_r) . In this context, a point (x, y) is defined to be local to another point if it lies within some distance R_q of it. Computation of these quadratics constitutes the main work done by this part of the routine. If there are less than 5 other points within distance R_q from (x_r, y_r) , the quadratic is replaced by a linear function. In cases of rank-deficiency, the minimum norm solution is computed.

The user may specify values for R_w and R_q , but it is usually easier to choose instead two integers N_w and N_q , from which the routine will compute R_w and R_q . These integers can be thought of as the average numbers of data points lying within distances R_w and R_q respectively from each node. Default values are provided, and advice on alternatives is given in Section 6.

The interpolant $F(x, y)$ generated by this routine can subsequently be evaluated for n points (x_k, y_k) , for $k = 1, 2, \dots, n$, in the domain of the data by a call to nag_2d_scatter_eval (e01sbc).

4. Parameters

method

Input: indicates the method to be used in interpolating the surface.

If **method** = **Nag_RC**, the method due to Renka and Cline is used.

If **method** = **Nag_Shep**, then a modification of Shepard's method is used.

Constraint: **method** = **Nag_RC** or **Nag_Shep**.

m

Input: the number of data points, m .

Constraint: $m \geq 3$.

x[m]

y[m]

f[m]

Input: the co-ordinates of the r th data point, for $r = 1, 2, \dots, m$. The data points are accepted in any order, but see Section 6.

Constraint: Each node must be unique. Additionally for **Nag_RC** method the (x, y) nodes must not be all collinear.

comm

Pointer to a communication structure of type Nag_Scatter_Struct. For **method** = **Nag_RC**, this structure contains the computed triangulation and the estimated partial derivatives at the nodes. For **method** = **Nag_Shep**, this structure contains the coefficients of the constructed nodal functions.

This structure must be passed **unchanged** to the interpolant evaluating function nag_2d_scatter_eval (e01sbc).

optional

Pointer to structure of type Nag_E01_Opt, which may be used only if **method** = **Nag_Shep**. It has the following members:

rnw – double

rnq – double

Input: suitable values for the radii R_w and R_q , described in Section 3.

Constraint: $0 < \text{optional.rnw} \leq \text{optional.rnq}$.

If this constraint is satisfied then these values are used, otherwise if **rnq** ≤ 0 then suitable values of R_w and R_q are computed from other members of this structure, **nw** and **nq**.

Output: **optional.rnw** and **optional.rnq** contain the actual values of R_w and R_q used by nag_2d_scatter_interpolant.

nw – Integer

nq – Integer

Input: these are integer values which can be used to compute the radii R_w and R_q , if the user does not wish to supply appropriate values directly, i.e., **optional.rnq** ≤ 0 .

Constraint: $0 < \text{optional.nw} \leq \text{optional.nq}$.

If this constraint is satisfied then nag_2d_scatter_interpolant computes the appropriate values of the radii R_w and R_q . Otherwise, if **optional.nq** ≤ 0 , then **optional.nw** and **optional.nq** are set to 9 and 18 respectively and these values are used to compute the radii R_w and R_q .

Output: If the values of **optional.rnw** and **optional.rnq** as supplied are used for the radii R_w and R_q then **optional.nw** and **optional.nq** are set to zero, otherwise **optional.nw** and **optional.nq** contain the actual values used for computing the radii R_w and R_q .

If this structure is supplied as a null pointer (`Nag_E01_Opt`)*0, also defined by Nag as `E01_DEFAULT` when **method** = **Nag_Shep**, then the default values of **nw** = 9 and **nq** = 18 are used to compute the radii.

minnq – Integer

Output: the minimum number of data points that lie within radius **optional.rnq** of any node, and thus define a nodal function. If **optional.minnq** is very small (say, less than 5), then the interpolant may be unsatisfactory in regions where the data points are sparse. `nag_2d_scatter_interpolant` generates a warning in this case.

Note that this structure is not referenced when **method** = **Nag_RC**. It should however be specified as `E01_DEFAULT`.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_BAD_PARAM

On entry, parameter **method** had an illegal value.

NE_INT_ARG_LT

On entry, **m** must not be less than 3: **m** = $\langle \text{value} \rangle$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_ALL_DATA_COLLINEAR

On entry, all the (**x**,**y**) pairs are collinear. Consider specifying **method** = **Nag_Shep** or using a one-dimensional interpolating function `nag_1d_spline_interpolant` (`e01bac`).

NE_DATA_NOT_UNIQUE

On entry, each data pair is not unique since data points (**x**[$\langle \text{value} \rangle$],**y**[$\langle \text{value} \rangle$]) and (**x**[$\langle \text{value} \rangle$],**y**[$\langle \text{value} \rangle$]) are identical and equal to ($\langle \text{value} \rangle$, $\langle \text{value} \rangle$).

NE_NUM_PARAM_INVALID

On entry, either or both of **optional.nq** and **optional.nw** are invalid, **optional.nq** = $\langle \text{value} \rangle$ and **optional.nw** = $\langle \text{value} \rangle$. **optional.nq** and **optional.nw** must satisfy the following constraints: $0 < \text{optional.nq} \leq \text{optional.nw}$.

NE_RAD_PARAM_INVALID

On entry, either or both of **optional.rnq** and **optional.rnw** are invalid, **optional.rnq** = $\langle \text{value} \rangle$ and **optional.rnw** = $\langle \text{value} \rangle$. **optional.rnq** and **optional.rnw** must satisfy the following constraints: $0.0 < \text{optional.rnw} \leq \text{optional.rnq}$.

NW_SPARSE_DATA_FIT

The minimum number of data points $\langle \text{value} \rangle$ that lie within the radius **optional.rnq** of any node is small enough to indicate that the interpolant may be unsatisfactory in regions where the data points are sparse. Current values of other relevant parameters (available as members of the structure **optional**, if this has been defined) are **rnq** = $\langle \text{value} \rangle$, **rnw** = $\langle \text{value} \rangle$, **nq** = $\langle \text{value} \rangle$, **nw** = $\langle \text{value} \rangle$.

6. Further Comments

If **method** = **Nag_RC**, the time taken for a call of `nag_2d_scatter_interpolant` is approximately proportional to the number of data points, m . The routine is more efficient if, before entry, the values in **x**, **y**, **f** are arranged so that the **x** array is in ascending order.

If **method** = **Nag_Shep**, the time taken for a call of `nag_2d_scatter_interpolant` is also approximately proportional to the number of data points, m , provided that N_q is of the same order as its default value (18). However, if N_q is increased so that the method becomes more global, the time taken becomes approximately proportional to m^2 .

In particular, when **method** = **Nag_Shep**, note first that the radii R_w and R_q , described in Section 3, are computed as $\frac{D}{2} \sqrt{\frac{N_w}{m}}$ and $\frac{D}{2} \sqrt{\frac{N_q}{m}}$ respectively, where D is the maximum distance between any pair of data points.

Default values $N_w = 9$ and $N_q = 18$ work quite well when the data points are fairly uniformly distributed. However, for data having some regions with relatively few points or for small data sets ($m < 25$), a larger value of N_w may be needed. This is to ensure a reasonable number of data points within a distance R_w of each node, and to avoid some regions in the data area being left outside all the discs of radius R_w on which the weights $w_r(x, y)$ are non-zero. Maintaining N_q approximately equal to $2N_w$ is usually an advantage.

Note however that increasing N_w and N_q does not improve the quality of the interpolant in all cases. It does increase the computational cost and makes the method less local.

At the end of the program, especially before any repeated call to nag_2d_scat_interpolant, the function nag_2d_scat_free (e01szc) must be called with the communication structure **comm** as the argument in order to free the memory previously allocated to the pointer members of **comm**.

6.1. Accuracy

On successful exit, the computational errors should be negligible in most situations but the user should always check the computed surface for acceptability, by drawing contours for instance. The surface always interpolates the input data exactly.

6.2. References

- Cline A K and Renka R L (1984) A Storage-efficient Method for Construction of a Thiessen Triangulation *Rocky Mountain J. Math.* **14** 119–139.
- Franke R and Nielson G (1980) Smooth Interpolation of Large Sets of Scattered Data *Internat. J. Num. Methods Engrg.* **15** 1691–1704.
- Lawson C L (1977) Software for C^1 Surface Interpolation *Mathematical Software III* (ed J R Rice) Academic Press pp 161–194.
- Renka R L (1984) Algorithm 624: Triangulation and Interpolation of Arbitrarily Distributed Points in the Plane *ACM Trans. Math. Softw.* **10** 440–442.
- Renka R L and Cline A K (1984) A Triangle-based C^1 Interpolation Method *Rocky Mountain J. Math.* **14** 223–237.
- Shepard D (1968) A Two-dimensional Interpolation Function for Irregularly Spaced Data *Proc. 23rd Nat. Conf. ACM*. Brandon/Systems Press Inc, Princeton pp 517–523.

7. See Also

nag_2d_scat_eval (e01sbc)
nag_2d_scat_free (e01szc)

8. Example

This program reads in a set of 30 data points and calls nag_2d_scat_interpolant to construct an interpolating surface. It then calls nag_2d_scat_eval (e01sbc) to evaluate the interpolant at a sample of points on a rectangular grid. The two methods described in Section 3 are used in the construction of the interpolating surface and the subsequent evaluation of the interpolant.

Note that this example is not typical of a realistic problem: the number of data points would normally be larger, and the interpolant would need to be evaluated on a finer grid to obtain an accurate plot, say.

8.1. Program Text

```
/* nag_2d_scat_interpolant(e01sac) Example Program
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nage01.h>

#define MMAX 100
```

```

#define NMAX 10
#define NXYM NMAX*NMAX
#define TDX MMAX
#define TDQ MMAX+1

main()
{
    Integer i, isel, j, m, n, nx, ny;
    double xhi, xlo, yhi, ylo;
    double f[MMAX], pf[NXYM], px[NXYM], py[NXYM], x[MMAX], y[MMAX];
    Nag_Scat_Struct comm;
    Nag_2d_Scat_Method method;
    Nag_E01_Opt optional;

    Vprintf("e01sac Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[\n]");
    /* Input the number of nodes. */
    Vscanf("%ld", &m);

    if (m>=1 && m<=MMAX)
    {
        /* Input the nodes (x,y) and heights, f. */
        for (i=0; i<m; ++i)
            Vscanf("%lf%lf%lf", &x[i], &y[i], &f[i]);

        /* Input the domain for evaluating the interpolant. */
        Vscanf("%ld%lf%lf", &nx, &xlo, &xhi);
        Vscanf("%ld%lf%lf", &ny, &ylo, &yhi);

        for (isel=1; isel<=2; ++isel)
        {
            /* Select the method of interpolation. */
            if (isel==1)
                method = Nag_RC;
            else if (isel==2)
                method = Nag_Shep;

            if (method==Nag_RC)
            {
                Vprintf("\nExample 1: Surface interpolation by method\
of Renka and Cline.\n\n");
                /*
                 * Generate the triangulation and gradients using the selected
                 * method.
                 */

                e01sac(method, m, x, y, f, &comm, (Nag_E01_Opt *)0,
                    NAGERR_DEFAULT);
            }
            else if (method==Nag_Shep)
            {
                Vprintf("\n\nExample 2: Surface interpolation by modified\
Shepard's method.\n\n");
                /* Compute the nodal function coefficients. */
                optional.nq = 24;
                optional.nw = 12;
                optional.rnq = -1.0;

                e01sac(method, m, x, y, f, &comm, &optional, NAGERR_DEFAULT);

                Vprintf("    optional.rnw =%8.2f    optional.rnq =%8.2f\n\n",
                    optional.rnw, optional.rnq);
                Vprintf("    minimum number of data points that lie within radius\
optional.rnq =%3ld\n", optional.minnq);
            }
            /*
             * Evaluate the interpolant on a rectangular grid at nx*ny points
             * over the domain (xlo to xhi) x (ylo to yhi).

```

```

    */
    n = 0;
    if (nx*ny<=NXYM)
    {
        for (j=0; j<ny; ++j)
        {
            for (i=0; i<nx; ++i)
            {
                px[i+nx*j] = ((double)(nx-i-1) / (nx-1)) * xlo +
                    ((double)(i) / (nx-1)) * xhi;
                py[i+nx*j] = ((double)(ny-j-1) / (ny-1)) * ylo +
                    ((double)(j) / (ny-1)) * yhi;
                ++n;
            }
        }
        if (method == Nag_RC)
            e01sbc(&comm, n, px, py, pf, NAGERR_DEFAULT);
        else if (method == Nag_Shep)
            e01sbc(&comm, n, px, py, pf, NAGERR_DEFAULT);

        Vprintf("\n          x");
        for (i = 0; i < nx; i++)
            Vprintf("%8.2f", px[i]);
        Vprintf("\n          y\n");
        for (i = ny-1; i >= 0; --i)
        {
            Vprintf("%8.2f  ", py[nx * i]);
            for (j = 0; j < nx; j++)
                Vprintf("%8.2f", pf[nx * i + j]);
            Vprintf("\n");
        }
    }
    /* Free the memory allocated to the pointers in structure comm. */
    e01szc(&comm);
}
exit(EXIT_SUCCESS);
}
else
{
    Vfprintf(stderr, "m is out of range: m = %-3ld\n", m);
    exit(EXIT_FAILURE);
}
}
/* main */

```

8.2. Program Data

e01sac Example Program Data

```

30
11.16    1.24    22.15
12.85    3.06    22.11
19.85   10.72     7.97
19.72    1.39   16.83
15.91    7.74   15.30
 0.00   20.00   34.60
20.87   20.00    5.74
 3.45   12.78   41.24
14.26   17.87   10.74
17.43    3.46   18.60
22.80   12.39    5.47
 7.58    1.98   29.87
25.00   11.87    4.40
 0.00    0.00   58.20
 9.66   20.00    4.73
 5.22   14.66   40.36
17.25   19.57    6.43
25.00    3.87    8.74
12.13   10.79   13.71
22.23    6.21   10.25
11.52    8.53   15.74
15.20    0.00   21.60
 7.54   10.69   19.31

```

17.32	13.78	12.11
2.14	15.03	53.10
0.51	8.37	49.43
22.69	19.63	3.25
5.47	17.13	28.63
21.67	14.36	5.52
3.31	0.33	44.08
7	3.0	21.0
6	2.0	17.0

8.3. Program Results

e01sac Example Program Results

Example 1: Surface interpolation by method of Renka and Cline.

	x	3.00	6.00	9.00	12.00	15.00	18.00	21.00
y								
17.00		41.25	27.62	18.03	12.29	11.68	9.09	5.37
14.00		47.61	36.66	22.87	14.02	13.44	11.20	6.46
11.00		38.55	25.25	16.72	13.83	13.08	10.71	6.88
8.00		37.90	23.97	16.79	16.43	15.46	13.02	9.30
5.00		40.49	29.26	22.51	20.72	19.30	16.72	12.87
2.00		43.52	33.91	26.59	22.23	21.15	18.67	14.88

Example 2: Surface interpolation by modified Shepard's method.

optional.rnw = 9.49 optional.rnq = 13.42

minimum number of data points that lie within radius optional.rnq = 7

	x	3.00	6.00	9.00	12.00	15.00	18.00	21.00
y								
17.00		40.23	27.72	21.23	14.59	12.00	9.43	5.46
14.00		46.96	37.37	23.74	14.67	13.25	11.29	6.26
11.00		39.42	25.42	16.32	13.78	12.60	10.39	7.03
8.00		37.50	22.36	18.57	15.63	15.55	13.05	9.69
5.00		41.25	31.76	24.74	21.17	18.93	16.83	12.65
2.00		44.58	34.35	26.47	22.27	20.98	18.69	15.06
