nag_real_cholesky (f03aec)

1. Purpose

nag_real_cholesky (f03aec) computes a Cholesky factorization of a real symmetric positive-definite matrix, and evaluates the determinant.

2. Specification

```
#include <nag.h>
#include <nagf03.h>

void nag_real_cholesky(Integer n, double a[], Integer tda, double p[],
     double *detf, Integer *dete, NagError *fail)
```

3. Description

This function computes the Cholesky factorization of a real symmetric positive-definite matrix $A = LL^T$ where L is lower triangular. The determinant is the product of the squares of the diagonal elements of L.

4. Parameters

n

Input: n, the order of the matrix A.
Constraint: $\mathbf{n} \geq 1$.

a[n][tda]

Input: the upper triangle of the n by n positive-definite symmetric matrix A. The elements of the array below the diagonal need not be set.
Output: the sub-diagonal elements of the lower triangular matrix L. The upper triangle of A is unchanged.

tda

Input: the second dimension of the array **a** as declared in the function from which nag_real_cholesky is called.
Constraint: $\mathbf{tda} \geq \mathbf{n}$.

p[n]

Output: the reciprocals of the diagonal elements of L.

detf
dete

Output: the determinant of A is given by $\mathbf{detf} \times 2.0^{\mathbf{dete}}$. It is given in this form to avoid overflow or underflow.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_NOT_POS_DEF

The matrix is not positive-definite, possibly due to rounding errors. The factorization could not be completed. **detf** and **dete** are set to zero.

NE_INT_ARG_LT

On entry, **n** must not be less than 1: $\mathbf{n} = \langle value \rangle$.

NE_2_INT_ARG_LT

On entry, $\mathbf{tda} = \langle value \rangle$ while $\mathbf{n} = \langle value \rangle$. These parameters must satisfy $\mathbf{tda} \geq \mathbf{n}$.

6. Further Comments

The time taken by the function is approximately proportional to n^3.

6.1. Accuracy

The accuracy of the determinant depends on the conditioning of the original matrix. For a detailed error analysis see Wilkinson and Reinsch (1971) p 25.

6.2. References

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation (Vol II, Linear Algebra)* Springer-Verlag pp 9–30.

7. See Also

nag_real_cholesky_solve_mult_rhs (f04agc)

8. Example

To compute a Cholesky factorization and calculate the determinant of the real symmetric positive-definite matrix

$$\begin{pmatrix} 6 & 7 & 6 & 5 \\ 7 & 11 & 8 & 7 \\ 6 & 8 & 11 & 9 \\ 5 & 7 & 9 & 11 \end{pmatrix}.$$

8.1. Program Text

```
/* nag_real_cholesky(f03aec) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <math.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagf03.h>

#define NMAX 8
#define TDA NMAX

main()
{
  double detf, determ, a[NMAX][TDA], p[NMAX];
  Integer i, dete, j, n;
  static NagError fail;

  Vprintf("f03aec Example Program Results\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vscanf("%ld\n",&n);
  if (n<1 || n>NMAX)
    {
      Vfprintf(stderr,"n is out of range: n = %5ld\n",n);
      exit(EXIT_FAILURE);
    }
  for (i=0; i<n; i++)
    for (j=0; j<n; j++)
      Vscanf("%lf",&a[i][j]);
  fail.print = TRUE;
  f03aec(n, (double *)a, (Integer)TDA, p, &detf, &dete, &fail);
  if (fail.code != NE_NOERROR)
    exit(EXIT_FAILURE);
  Vprintf("Array A after factorization\n");
  for (i=0; i<n; i++)
    for (j=0; j<n; j++)
      Vprintf("%9.4f%s", a[i][j], (j%8==7 || j==n-1) ? "\n" : " ");
```

```
  Vprintf("\nArray p\n");
  for (i=0; i<n; i++)
    Vprintf("%9.4f%s", p[i], (i%8==7 || i==n-1) ? "\n" : " ");
  Vprintf("\ndetf = %9.4f           dete = %2ld\n\n", detf, dete);
  determ = detf*pow(2.0,(double)dete);
  Vprintf("Value of determinant = %9.4f\n", determ);
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
f03aec Example Program Data
  4
    6    7    6    5
    7   11    8    7
    6    8   11    9
    5    7    9   11
```

8.3. Program Results

```
f03aec Example Program Results
Array A after factorization
   6.0000    7.0000    6.0000    5.0000
   2.8577   11.0000    8.0000    7.0000
   2.4495    0.5941   11.0000    9.0000
   2.0412    0.6931    1.6645   11.0000

Array p
   0.4082    0.5941    0.4639    0.5283

detf =    0.0691           dete = 12

Value of determinant =  283.0000
```