nag_deviates_normal_dist (g01cec)

1. Purpose

nag_deviates_normal_dist (g01cec) returns the deviate, x_p , associated with the given lower tail probability, p, of the standardised Normal distribution.

2. Specification

#include <nag.h>
#include <nagg01.h>

double nag_deviates_normal_dist(double p, NagError *fail)

3. Description

 x_p is calculated for the given p such that

$$p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x_p} e^{-u^2/2} \, du \qquad -\infty < x_p < \infty.$$

The method used is an extension of that of Beasley and Springer (1977). p is first replaced by q = p - 0.5.

(a) If $|q| \leq 0.3$, x_p is computed by a rational Chebyshev approximation

$$x_p = s \frac{A(s^2)}{B(s^2)}$$

where $s = \sqrt{2\pi q}$ and A, B are polynomials of degree 7.

(b) If $0.3 < |q| \le 0.42$, x_p is computed by a rational Chebyshev approximation

$$x_p = \operatorname{sign} q\left(\frac{C(t)}{D(t)}\right)$$

where t = |q| - 0.3 and C, D are polynomials of degree 5.

(c) If |q| > 0.42, x_p is computed as

$$x_p = \operatorname{sign} q\left(\frac{E(u)}{F(u)}\right) + u$$

where $u = \sqrt{-2\log(\min(p, 1-p))}$ and E, F are polynomials of degree 6.

4. Parameters

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Input: the probability, p, from the standardised Normal distribution. Constraint: $0.0 < \mathbf{p} < 1.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LE

On entry, **p** must not be less than or equal to 0.0: $\mathbf{p} = \langle value \rangle$.

NE_REAL_ARG_GE

On entry, **p** must not be greater than or equal to 1.0: $\mathbf{p} = \langle value \rangle$.

6. Further Comments

If X is a Normal random variable with mean μ and variance σ^2 , the deviate corresponding to a lower tail probability of p is $\mu + \sigma x_p$, where x_p is the standardised Normal deviate returned by nag_deviates_normal_dist.

6.1. Accuracy

The function attempts to attain a relative precision of 5.0×10^{-13} .

6.2. References

Beasley J D and Springer S G (1977) Algorithm AS111: The percentage points of the Normal distribution Appl. Stat. 26 118–120.

7. See Also

nag_cumul_normal (s15abc) nag_cumul_normal_complem (s15acc)

8. Example

The deviates corresponding to several lower tail probabilities from the standard Normal distribution are calculated and printed.

8.1. Program Text

```
/* nag_deviates_normal_dist(g01cec) Example Program
      * Copyright 1990 Numerical Algorithms Group.
      * Mark 2 revised, 1992.
      */
     #include <nag.h>
     #include <stdio.h>
     #include <nag_stdlib.h>
     #include <nagg01.h>
     #define NMAX 5
     main()
     ſ
       double x;
       static double p[] = {0.95, 0.5, 0.995, 0.75, 0.001};
       Integer i;
       Vprintf("g01cec Example Program Results\n");
Vprintf(" Prob. Deviate\n\n");
for (i=0; i<NMAX; i++)</pre>
          ſ
            x = g01cec(p[i], NAGERR_DEFAULT);
            Vprintf("%7.3f%11.4f\n", p[i], x);
          }
       exit(EXIT_SUCCESS);
     }
8.2. Program Data
```

None.

```
8.3. Program Results
```

gO1cec Example Program Results Prob. Deviate

0.950	1.6449
0.500	0.0000
0.995	2.5758
0.750	0.6745
0.001	-3.0902