

## nag\_prob\_students\_t (g01ebc)

### 1. Purpose

**nag\_prob\_students\_t (g01ebc)** returns the lower tail, upper tail or two-tail probability for the Student's  $t$ -distribution with real degrees of freedom.

### 2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_students_t(Nag_TailProbability tail, double t, double df,
                          NagError *fail)
```

### 3. Description

The lower tail probability for the Student's  $t$ -distribution with  $\nu$  degrees of freedom,  $P(T \leq t : \nu)$ , is defined by

$$P(T \leq t : \nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)} \int_{-\infty}^t \left[1 + \frac{T^2}{\nu}\right]^{-(\nu+1)/2} dT, \quad \nu \geq 1.$$

Computationally, there are two situations:

- (a) when  $\nu < 20$ , a transformation of the beta distribution,  $P_\beta(B \leq \beta : a, b)$  is used;

$$P(T \leq t : \nu) = \frac{1}{2}P_\beta\left(B \leq \frac{\nu}{\nu+t^2} : \nu/2, \frac{1}{2}\right) \quad \text{when } t < 0.0$$

or

$$P(T \leq t : \nu) = \frac{1}{2} + \frac{1}{2}P_\beta\left(B \geq \frac{\nu}{\nu+t^2} : \nu/2, \frac{1}{2}\right) \quad \text{when } t > 0.0$$

- (b) when  $\nu \geq 20$ , an asymptotic normalising expansion of the Cornish–Fisher type is used to evaluate the probability, see Hill (1970).

### 4. Parameters

#### tail

Input: indicates which tail the returned probability should represent.

If **tail** = **Nag\_UpperTail**, the upper tail probability is returned, i.e.,  $P(T \geq t : \nu)$ .

If **tail** = **Nag\_LowerTail**, the lower tail probability is returned, i.e.,  $P(T \leq t : \nu)$ .

If **tail** = **Nag\_TwoTailSignif**, the two tail (significance level) probability is returned, i.e.,  $P(T \geq |t| : \nu) + P(T \leq -|t| : \nu)$ .

If **tail** = **Nag\_TwoTailConfid**, the two tail (confidence interval) probability is returned, i.e.,  $P(T \leq |t| : \nu) - P(T \leq -|t| : \nu)$ .

Constraint: **tail** = **Nag\_UpperTail** or **Nag\_LowerTail** or **Nag\_TwoTailSignif** or **Nag\_TwoTailConfid**.

#### t

Input: the value of the Student's  $t$  variate,  $t$ .

#### df

Input: the degrees of freedom,  $\nu$ , of the Student's  $t$ -distribution.

Constraint: **df**  $\geq 1$ .

#### fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

On any of the error conditions listed below nag\_prob\_students\_t returns 0.0.

### NE\_BAD\_PARAM

On entry, parameter **tail** had an illegal value.

### NE\_REAL\_ARG\_LT

On entry, **df** must not be less than 1.0: **df** =  $\langle \text{value} \rangle$ .

## 6. Further Comments

The probabilities could also be obtained by using the appropriate transformation to a Beta distribution (see Abramowitz and Stegun, 1965) and using nag\_prob\_beta\_dist (g01eec). This function allows the user to set the required accuracy.

### 6.1. Accuracy

The computed probability should be accurate to 5 significant places for reasonable probabilities but there will be some loss of accuracy for very low probabilities (less than  $10^{-10}$ ), see Hill (1970).

### 6.2. References

Abramowitz M and Stegun I A (1965) *Handbook of Mathematical Functions* Dover Publications, New York ch 26.

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

Hill G W (1970) Student's *t*-distribution *Commun. ACM* **13** (10) 617–619.

## 7. See Also

None.

## 8. Example

Values from, and degrees of freedom for Student's *t*-distributions are read along with the required tail. The probabilities are calculated and printed until the end of data is reached.

### 8.1. Program Text

```
/* nag_prob_students_t(g01ebc) Example Program
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
    double df, prob, t;
    int i;
    static Nag_TailProbability tail[4] = {Nag_LowerTail, Nag_UpperTail,
                                          Nag_TwoTailSignif, Nag_TwoTailConfid};
    static char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
                                "Nag_TwoTailSignif", "Nag_TwoTailConfid"};

    Vprintf("g01ebc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("      t      df      prob      tail\n\n");
    while (scanf("%lf %lf %ld\\n", &t, &df, &i) != EOF)
    {
        prob = g01ebc(tail[i], t, df, NAGERR_DEFAULT);
        Vprintf(" %6.3f%8.3f%8.4f %s\\n", t, df, prob, tailmess[i]);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01ebc Example Program Data
0.85  20.0  0
0.85  20.0  2
0.85  20.0  3
0.85  20.0  1
```

8.3. Program Results

```
g01ebc Example Program Results

      t      df      prob      tail
0.850  20.000  0.7973  Nag_LowerTail
0.850  20.000  0.4054  Nag_TwoTailSignif
0.850  20.000  0.5946  Nag_TwoTailConfid
0.850  20.000  0.2027  Nag_UpperTail
```

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