nag_prob_beta_dist (g01eec)

1. Purpose

nag_prob_beta_dist (g01eec) computes the upper and lower tail probabilities and the probability density function of the beta distribution with parameters a and b.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

void nag_prob_beta_dist(double x, double a, double b, double tol, double *p,
     double *q, double *pdf, NagError *fail)
```

3. Description

The probability density function of the beta distribution with parameters a and b is

$$f(B:a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} B^{a-1}(1-B)^{b-1} \qquad 0 \le B \le 1;\ a,b > 0.$$

The lower tail probability, $P(B \le \beta : a,b)$ is defined by

$$P(B \le \beta : a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^\beta B^{a-1}(1-B)^{b-1}\,dB = I_\beta(a,b) \qquad 0 \le \beta \le 1; a,b > 0.$$

The function $I_x(a,b)$ is also known as the incomplete beta function.

The method used is similar to that described by Majumder and Bhattacharjee (1973), and uses the following three relations for the incomplete beta function (see Abramowitz and Stegun, 1972)

$$\begin{array}{rcll} I_x(a,b) & = & \frac{\Gamma(a+b)}{\Gamma(a+1)\Gamma(b)} x^a(1-x)^{b-1} + I_x(a+1,b-1) & (1) \\ I_x(a,b) & = & \frac{\Gamma(a+b)}{\Gamma(a+1)\Gamma(b)} x^a(1-x)^{b} + I_x(a+1,b) & (2) \\ I_x(a,b) & = & 1 - I_{1-x}(b,a) & (3) \end{array}$$

If a is less than $(a+b)x$, then a and b are interchanged and $(1-x)$ replaces x, with relation (3) being used to obtain the final result.

Relation (1) is applied repeatedly until the second parameter is reduced to b', where $0 < b' \le 1$. This produces a power series of finite length, in $x/(1-x)$, whose sum is found. If $b' = 1$, this sum equals $I_x(a,b)$, since $I_x(c,1) = x^c/c$ for all $c > 0$.

Otherwise ($b' \ne 1$), the integral $I_x(c,d)$ remains to be evaluated, where

$$c = a+b-b' \qquad d = b' \qquad 0 < b' < 1.$$

Relation (2) applied repeatedly gives a convergent power series in x of infinite length.

4. Parameters

x

Input: the value of the beta variate, β.
Constraint: $0.0 \le \mathbf{x} \le 1.0$.

a

Input: the first parameter, a, of the required beta distribution.
Constraint: $0.0 < \mathbf{a} \le 10^6$.

b

Input: the second parameter, b, of the required beta distribution.
Constraint: $0.0 < \mathbf{b} \le 10^6$.

tol

Input: the relative accuracy required by the user in the results. If nag_prob_beta_dist is entered with **tol** greater than or equal to 1.0 or less than 10 times the ***machine precision***, then the value of 10 times ***machine precision*** is used instead.

p

Output: the lower tail probability, $P(B \leq \beta : a, b)$.

q

Output: the upper tail probability, $P(B \geq \beta : a, b)$.

pdf

Output: the probability density function, $f(B : a, b)$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LT

On entry, **x** must not be less than 0.0: **x** = ⟨*value*⟩.

NE_REAL_ARG_GT

On entry, **x** must not be greater than 1.0: **x** = ⟨*value*⟩.
On entry, **a** must not be greater than 10^6: **a** = ⟨*value*⟩.
On entry, **b** must not be greater than 10^6: **b** = ⟨*value*⟩.

NE_REAL_ARG_LE

On entry, **a** must not be less than or equal to 0.0: **a** = ⟨*value*⟩.
On entry, **b** must not be less than or equal to 0.0: **b** = ⟨*value*⟩.

NE_RES_NOT_ACC

The requested accuracy has not been achieved. Use a larger value of **tol**.
The values returned for **p** and **q** should be reasonable approximations.

NE_PROBAB_CLOSE_TO_TAIL

The probability is too close to 0.0 or 1.0.
The result returned is either 0 or 1 as appropriate. This should be a good approximation to the required solution.

6. Further Comments

The time taken by the function will depend on the shape of the distribution. For highly skewed distributions with one of the values of a, b large and the other small, series (2) will take longer to converge than for distributions which are more symmetric.

6.1. Accuracy

The convergence of series (2) is assumed when an upper bound on the sum of the remaining terms is less than **tol**. Summation also ceases if the relative change in the sum of the series is less than ***machine precision***, in which case full accuracy cannot be guaranteed.

The accuracy is limited by the error in evaluating the logarithm of the gamma function.

6.2. References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications, New York p 944.

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

Majumder K L and Bhattacharjee G P (1973) The incomplete beta integral *Appl. Stat.* **22** Algorithm AS63 409–411.

7. See Also

None.

8. Example

Values for several beta distributions are read, and the probabilities calculated and printed, until the end of data is reached.

8.1. Program Text

```
/* nag_prob_beta_dist(g01eec) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
  double a, b, p, pdf, q, tol, x;
  static NagError fail;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("g01eec Example Program Results\n");
  Vprintf("   x         a            b            p            q\
        pdf\n\n");
  while (scanf("%lf %lf %lf %lf", &x, &a, &b, &tol) != EOF)
    {
      g01eec(x, a, b, tol, &p, &q, &pdf, &fail);
      if (fail.code==NE_NOERROR)
        Vprintf("%7.4f%13.4e%13.4e%13.4e%13.4e%13.4e\n", x, a, b, p, q, pdf);
      else
        {
          Vprintf("%7.4f%13.4e%13.4e%13.4e%13.4e%13.4e\n Note: %s\n",x,a,
                  b, p, q, pdf, fail.message);
          exit(EXIT_FAILURE);
        }
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01eec Example Program Data
 0.25  1.0  2.0  1.9
 0.75  1.5  1.5  0.0001
 0.5   2.0  1.0  1.01
```

8.3. Program Results

```
g01eec Example Program Results
   x         a            b            p            q           pdf

 0.2500   1.0000e+00   2.0000e+00   4.3750e-01   5.6250e-01   1.5000e+00
 0.7500   1.5000e+00   1.5000e+00   8.0450e-01   1.9550e-01   1.1027e+00
 0.5000   2.0000e+00   1.0000e+00   2.5000e-01   7.5000e-01   1.0000e+00
```