

**nag\_gamma\_dist (g01efc)****1. Purpose**

**nag\_gamma\_dist (g01efc)** returns the lower or upper tail probability of the gamma distribution, with parameters  $\alpha$  and  $\beta$ .

**2. Specification**

```
#include <nag.h>
#include <nagg01.h>
```

```
double nag_gamma_dist(Nag_TailProbability tail, double g, double a, double b,
    NagError *fail)
```

**3. Description**

The lower tail probability for the gamma distribution with parameters  $\alpha$  and  $\beta$ ,  $P(G \leq g)$ , is defined by

$$P(G \leq g; \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^g G^{\alpha-1} e^{-G/\beta} dG \quad \alpha > 0.0, \beta > 0.0.$$

The mean of the distribution is  $\alpha\beta$  and its variance is  $\alpha\beta^2$ . The transformation  $Z = G/\beta$  is applied to yield the following incomplete gamma function in normalised form,

$$P(G \leq g; \alpha, \beta) = P(Z \leq g/\beta : \alpha, 1.0) = \frac{1}{\Gamma(\alpha)} \int_0^{g/\beta} Z^{\alpha-1} e^{-Z} dZ.$$

This is then evaluated using **nag\_incomplete\_gamma (s14bac)**.

**4. Parameters****tail**

Input: indicates whether the upper or lower tail probability is required.

If **tail** = **Nag\_LowerTail**, the lower tail probability is returned, i.e.,  $P(G \leq g : \alpha, \beta)$ .

If **tail** = **Nag\_UpperTail**, the upper tail probability is returned, i.e.,  $P(G \geq g : \alpha, \beta)$ .

Constraint: **tail** = **Nag\_LowerTail** or **Nag\_UpperTail**.

**g**

Input: the value of the gamma variate,  $g$ .

Constraint: **g**  $\geq$  0.0.

**a**

Input: the parameter  $\alpha$  of the gamma distribution.

Constraint: **a**  $>$  0.0.

**b**

Input: the parameter  $\beta$  of the gamma distribution.

Constraint: **b**  $>$  0.0.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

**5. Error Indications and Warnings**

On any of the error conditions listed below except **NE\_ALG\_NOT\_CONV** **nag\_gamma\_dist** returns 0.0.

**NE\_BAD\_PARAM**

On entry, parameter **tail** had an illegal value.

**NE\_REAL\_ARG\_LT**

On entry, **g** must not be less than 0.0: **g** =  $\langle value \rangle$ .

**NE\_REAL\_ARG\_LE**

On entry, **a** must not be less than or equal to 0.0: **a** =  $\langle value \rangle$ .

On entry, **b** must not be less than or equal to 0.0: **b** =  $\langle value \rangle$ .

**NE\_ALG\_NOT\_CONV**

The algorithm has failed to converge in  $\langle value \rangle$  iterations.

The probability returned should be a reasonable approximation to the solution.

**6. Further Comments**

The time taken by the function varies slightly with the input parameters  $g$ ,  $\alpha$  and  $\beta$ .

**6.1. Accuracy**

The result should have a relative accuracy of *machine precision*. There are rare occasions when the relative accuracy attained is somewhat less than *machine precision* but the error should not exceed more than 1 or 2 decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in nag\_incomplete\_gamma (s14bac) are given to this precision.

**6.2. References**

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

**7. See Also**

nag\_incomplete\_gamma (s14bac)

**8. Example**

The following example reads in values for several gamma distributions, computes and prints the lower probabilities for each case, until the end of data is reached.

**8.1. Program Text**

```
/* nag_gamma_dist(g01efc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
    double a, b, g, p;

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("g01efc Example Program Results\\n");
    Vprintf(" Gamma deviate      Alpha      Beta      Lower tail prob.\\n\\n");
    while (scanf("%lf %lf %lf", &g, &a, &b) != EOF)
    {
        p = g01efc(Nag_LowerTail, g, a, b, NAGERR_DEFAULT);
        Vprintf(" %9.2f%13.2f%9.2f%14.4f\\n", g, a, b, p);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01efc Example Program Data
15.5  4.0  2.0
 0.5  4.0  1.0
10.0  1.0  2.0
 5.0  2.0  2.0
```

8.3. Program Results

```
g01efc Example Program Results
Gamma deviate      Alpha      Beta      Lower tail prob.
15.50              4.00      2.00      0.9499
 0.50              4.00      1.00      0.0018
10.00              1.00      2.00      0.9933
 5.00              2.00      2.00      0.7127
```

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