NAG C Library Function Document

nag_prob_von_mises (g01erc)

1 Purpose

nag_prob_von_mises (g01erc) returns the probability associated with the lower tail of the von Mises distribution between $-\pi$ and π.

2 Specification

```
double nag_prob_von_mises (double t, double vk, NagError *fail)
```

3 Description

The von Mises distribution is a symmetric distribution used in the analysis of circular data. The lower tail area of this distribution on the circle with mean direction $\mu_0 = 0$ and concentration parameter kappa, κ, can be written as

$$\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2\pi I_0(\kappa)} \int_{-\pi}^{\theta} e^{\kappa \cos \Theta} \mathrm{d}\Theta,$$

where θ is reduced modulo 2π so that $-\pi \leq \theta < \pi$ and $\kappa \geq 0$. Note that if $\theta = \pi$ then nag_prob_von_mises (g01erc) returns a probability of 1. For very small κ the distribution is almost the uniform distribution, whereas for $\kappa \to \infty$ all the probability is concentrated at one point.

The method of calculation for small κ involves backwards recursion through a series expansion in terms of modified Bessel functions, while for large κ an asymptotic Normal approximation is used.

In the case of small κ the series expansion of $\Pr(\Theta \leq \theta\text{: } \kappa)$ can be expressed as

$$\Pr(\Theta \leq \theta : \kappa) = \tfrac{1}{2} + \frac{\theta}{(2\pi)} + \frac{1}{\pi I_0(\kappa)} \sum_{n=1}^{\infty} n^{-1} I_n(\kappa) \sin n\theta,$$

where $I_n(\kappa)$ is the modified Bessel function. This series expansion can be represented as a nested expression of terms involving the modified Bessel function ratio R_n,

$$R_n(\kappa) = \frac{I_n(\kappa)}{I_{n-1}(\kappa)}, \quad n = 1, 2, 3, \ldots,$$

which is calculated using backwards recursion.

For large values of κ (see Section 7) an asymptotic Normal approximation is used. The angle Θ is transformed to the nearly Normally distributed variate Z,

$$Z = b(\kappa) \sin \tfrac{\Theta}{2},$$

where

$$b(\kappa) = \frac{\sqrt{\frac{2}{\pi}} e^{\kappa}}{I_0(\kappa)}$$

and $b(\kappa)$ is computed from a continued fraction approximation. An approximation to order κ^{-4} of the asymptotic normalizing series for z is then used. Finally the Normal probability integral is evaluated.

For a more detailed analysis of the methods used see Hill (1977).

4 References

Mardia K V (1972) *Statistics of Directional Data* Academic Press

Hill G W (1977) Algorithm 518: Incomplete Bessel function I_0: The Von Mises distribution *ACM Trans. Math. Software* **3** 279–284

5 Parameters

1: **t** – double *Input*

On entry: the observed von Mises statistic, θ, measured in radians.

2: **vk** – double *Input*

On entry: the concentration parameter κ, of the von Mises distribution.

Constraint: **vk** ≥ 0.

3: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_REAL

On entry, **vk** $= \langle value \rangle$.
Constraint: **vk** ≥ 0.0.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

nag_prob_von_mises (g01erc) uses one of two sets of constants depending on the value of ***machine precision***. One set gives an accuracy of six digits and uses the Normal approximation when **vk** ≥ 6.5, the other gives an accuracy of 12 digits and uses the Normal approximation when **vk** ≥ 50.

8 Further Comments

Using the series expansion for small κ the time taken by nag_prob_von_mises (g01erc) increases linearly with κ; for larger κ, for which the asymptotic Normal approximation is used, the time taken is much less.

If angles outside the region $-\pi \leq \theta < \pi$ are used care has to be taken in evaluating the probability of being in a region $\theta_1 \leq \theta \leq \theta_2$ if the region contains an odd multiple of π, $(2n+1)\pi$. The value of $F(\theta_2; \kappa) - F(\theta_1; \kappa)$ will be negative and the correct probability should then be obtained by adding one to the value.

9 Example

Four values from the von Mises distribution along with the values of the parameter κ are input and the probabilities computed and printed.

9.1 Program Text

```
/* nag_prob_von_mises (g01erc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{

  /* Scalars */
  double p, t, vk;
  Integer exit_status, i__, n;
  NagError fail;

  INIT_FAIL(fail);
  exit_status = 0;

  Vprintf("%s\n\n", "g01erc Example Program Results");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%*[^\n] ",   &n);

  for (i__ = 1; i__ <= n; ++i__)
    {

      Vscanf("%lf%lf%*[^\n] ",   &t,  &vk);
      p = g01erc(t, vk, &fail);
      if (fail.code == NE_NOERROR)
        {
          Vprintf(" p = %10.4f\n",   p);
        }
      else
        {
          Vprintf("Error from g01erc.\n%s\n",   fail.message);
          exit_status = 1;
          goto END;
        }
    }
 END:
  return exit_status;
}
```

9.2 Program Data

```
g01erc Example Program Data
 4
 7.0  0.0
 2.8  2.4
 1.0  1.0
-1.4  1.3
```

9.3 Program Results

```
g01erc Example Program Results

 p =     0.6141
 p =     0.9983
 p =     0.7944
 p =     0.1016
```