# nag\_deviates\_students\_t (g01fbc)

## 1. Purpose

 $nag_deviates_students_t$  (g01fbc) returns the deviate associated with the given tail probability of Student's *t*-distribution with real degrees of freedom.

# 2. Specification

#include <nag.h>
#include <nagg01.h>

## 3. Description

The deviate,  $t_p$  associated with the lower tail probability, p, of the Student's t-distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(T \le t_p : \nu) = p = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT \qquad \nu \ge 1; \ -\infty < t_p < \infty.$$

For  $\nu = 1$  or 2 the integral equation is easily solved for  $t_p$ .

For other values of  $\nu < 3$  a transformation to the beta distribution is used and the result obtained from nag\_deviates\_beta (g01fec).

For  $\nu \geq 3$  an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

## 4. Parameters

tail

Input: indicates which tail the supplied probability represents. If tail = Nag\_UpperTail, the upper tail probability, i.e.,  $P(T \ge t_p : \nu)$ . If tail = Nag\_LowerTail, the lower tail probability, i.e.,  $P(T \le t_p : \nu)$ . If tail = Nag\_TwoTailSignif, the two tail (significance level) probability, i.e.,  $P(T \ge |t_p| : \nu) + P(T \le -|t_p| : \nu)$ . If tail = Nag\_TwoTailConfid, the two tail (confidence interval) probability, i.e.,  $P(T \le |t_p| : \nu) - P(T \le -|t_p| : \nu)$ .

 $\label{eq:constraint: tail = Nag_UpperTail or Nag_LowerTail or Nag_TwoTailSignif or Nag_TwoTailConfid.$ 

#### р

Input: the probability, p, from the required Student's *t*-distribution as defined by **tail**. Constraint:  $0.0 < \mathbf{p} < 1.0$ .

### df

Input: the degrees of freedom,  $\nu$ , of the Student's *t*-distribution. Constraint:  $\mathbf{df} \geq 1.0$ .

#### fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

On any of the error conditions listed below except **NE\_SOL\_NOT\_CONV** nag\_deviates\_students\_t returns 0.0.

## NE\_BAD\_PARAM

On entry, parameter tail had an illegal value.

# NE\_REAL\_ARG\_LE

On entry, **p** must not be less than or equal to 0.0:  $\mathbf{p} = \langle value \rangle$ .

## NE\_REAL\_ARG\_GE

On entry, **p** must not be greater than or equal to 1.0:  $\mathbf{p} = \langle value \rangle$ .

### NE\_REAL\_ARG\_LT

On entry, **df** must not be less than 1.0:  $\mathbf{df} = \langle value \rangle$ .

## NE\_SOL\_CLOSE\_TO\_ZERO

The solution is too close to zero to be determined accurately. This error will only occur when  $\mathbf{df} = 1.0$ . The returned value of zero will be a good approximation in terms of absolute value but will have a poor relative precision.

## NE\_SOL\_NOT\_CONV

The solution has failed to converge. However, the result should be a reasonable approximation.

## 6. Further Comments

The value  $t_p$  may be calculated by using the transformation described in Section 3 and using nag\_deviates\_beta (g01fec). This function allows the user to set the required accuracy.

### 6.1. Accuracy

The results should be accurate to 5 significant digits, for most parameter values. The error behaviour for various parameter values is discussed in Hill (1970).

### 6.2. References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth. Hill G W (1970) Student's t quantiles *Commun. ACM* **13** (10) Algorithm 396 619–620.

## 7. See Also

 $nag_deviates_beta (g01fec)$ 

## 8. Example

Lower tail probabilities are read for several t-distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

#### 8.1. Program Text

```
/* nag_deviates_students_t(g01fbc) Example Program
 * Copyright 1996 Numerical Algorithms Group.
 * Mark 4, 1996.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>
main()
ſ
  double df, p, t;
  int i;
  static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
  Nag_TwoTailSignif, Nag_TwoTailConfid};
static char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
                                  "Nag_TwoTailSignif", "Nag_TwoTailConfid"};
  static NagError fail;
  Vprintf("g01fbc Example Program Results\n\n");
  /* Skip heading in data file */
```

8.2. Program Data

g01fbc Example Program Data 0.0100 20.0 2 0.01 7.5 0 0.99 45.0 3

## 8.3. Program Results

g01fbc Example Program Results

р	df	tail	t
0.010 0.010	20.000 7.500	Nag_TwoTailSignif Nag_LowerTail	2.845 -2.943
0.990	45.000	Nag_TwoTailConfid	2.690