

# NAG C Library Function Document

## nag\_tsa\_diff (g13aac)

### 1 Purpose

nag\_tsa\_diff (g13aac) carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

### 2 Specification

```
void nag_tsa_diff (const double x[], Integer nx, Integer d, Integer ds, Integer s,
                  double xd[], Integer *nxd, NagError *fail)
```

### 3 Description

Let  $\nabla^d \nabla_s^D x_i$  be the  $i$ th value of a time series  $x_i$ , for  $i = 1, 2, \dots, n$  after non-seasonal differencing of order  $d$  and seasonal differencing of order  $D$  (with period or seasonality  $s$ ). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i & d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i & D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order  $d$  is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i & \text{for } i = 1, 2, \dots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i & \text{for } i = 1, 2, \dots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i & \text{for } i = 1, 2, \dots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order  $D$  is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i & \text{for } i = 1, 2, \dots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i & \text{for } i = 1, 2, \dots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D+1} x_{i+s} - \nabla^d \nabla_s^{D+1} x_i & \text{for } i = 1, 2, \dots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of  $m = n - d - D \times s$  values.

### 4 References

None.

### 5 Parameters

- 1: **x[nx]** – const double *Input*  
*On entry:* the undifferenced time series,  $x_i$ , for  $i = 1, 2, \dots, n$ .
- 2: **nx** – Integer *Input*  
*On entry:* the number of values,  $n$ , in the undifferenced time series.  
*Constraint:* **nx** > **d** + (**ds** × **s**).

- 3: **d** – Integer *Input*  
*On entry:* the order of non-seasonal differencing,  $d$ .  
*Constraint:*  $d \geq 0$ .
- 4: **ds** – Integer *Input*  
*On entry:* the order of seasonal differencing,  $D$ .  
*Constraint:*  $ds \geq 0$ .
- 5: **s** – Integer *Input*  
*On entry:* the seasonality,  $s$ .  
*Constraints:*  
     if  $ds > 0$ ,  $s > 0$ ;  
     if  $ds = 0$ ,  $s \geq 0$ .
- 6: **xd[nx]** – double *Output*  
*On exit:* the differenced values in elements 1 to **nxd**, and reconstitution data in the remainder of the array.
- 7: **nxd** – Integer \* *Output*  
*On exit:* the number of differenced values in the array **xd**.
- 8: **fail** – NagError \* *Input/Output*  
 The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **s** =  $\langle value \rangle$ .

Constraint:  $s \geq 0$ .

On entry, **ds** =  $\langle value \rangle$ .

Constraint:  $ds \geq 0$ .

On entry, **d** =  $\langle value \rangle$ .

Constraint:  $d \geq 0$ .

### NE\_INT\_2

On entry, **s** = 0 and **ds** > 0: **ds** =  $\langle value \rangle$ .

### NE\_INT\_4

On entry,  $nx \leq d + (ds \times s)$ : **nx** =  $\langle value \rangle$ , **d** =  $\langle value \rangle$  **ds** =  $\langle value \rangle$ , **s** =  $\langle value \rangle$ .

### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

The computations are believed to be stable.

## 8 Further Comments

The time taken by the routine is approximately proportional to  $(d + ds) \times nx$ .

## 9 Example

The example program reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

### 9.1 Program Text

```
/* nag_tsa_diff (g13aac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    Integer exit_status, i, d, ds, s, nx, nxd;
    NagError fail;

    /* Arrays */
    double *x = 0, *xd = 0;

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("g13aac Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");

    Vscanf("%ld%ld%ld%ld%*[\n] ", &nx, &d, &ds, &s);

    if (nx > 0)
    {
        /* Allocate memory */
        if ( !(x = NAG_ALLOC(nx, double)) ||
            !(xd = NAG_ALLOC(nx, double)) )
        {
            Vprintf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }

        for (i = 1; i <= nx; ++i)
            Vscanf("%lf", &x[i-1]);
        Vscanf("%*[\n] ");

        Vprintf("\n");
        Vprintf("Non-seasonal differencing of order %ld "
            "and seasonal differencing\nof order %ld "
            "with seasonality %ld are applied\n", d, ds, s);
    }
}
```

```

g13aac(x, nx, d, ds, s, xd, &nxd, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g13aac.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

Vprintf("\n");
Vprintf("The output array holds %2ld values, of which the "
        "first %2ld are differenced values\n\n", nx, nxd);

for (i = 1; i <= nx; ++i)
{
    Vprintf("%10.1f", xd[i-1]);
    if (i % 5 == 0 || i == nx)
        Vprintf("\n");
}

END:
if (x) NAG_FREE(x);
if (xd) NAG_FREE(xd);

return exit_status;
}

```

## 9.2 Program Data

g13aac Example Program Data

```

20 2 1 4
120.0 108.0 98.0 118.0 135.0
131.0 118.0 125.0 121.0 100.0
82.0 82.0 89.0 88.0 86.0
96.0 108.0 110.0 99.0 105.0

```

## 9.3 Program Results

g13aac Example Program Results

Non-seasonal differencing of order 2 and seasonal differencing of order 1 with seasonality 4 are applied

The output array holds 20 values, of which the first 14 are differenced values

-11.0	-10.0	-8.0	4.0	12.0
-2.0	18.0	9.0	-4.0	-6.0
-5.0	-2.0	-12.0	5.0	2.0
-10.0	-13.0	17.0	6.0	105.0

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