NAG C Library Function Document

nag_tsa_cross_corr (g13bcc)

1 Purpose

nag_tsa_cross_corr (g13bcc) calculates cross-correlations between two time series.

2 Specification

```
void nag_tsa_cross_corr (const double x[], const double y[], Integer nxy,
     Integer nl, double *s, double *r0, double r[], double *stat, NagError *fail)
```

3 Description

Given two series $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ the routine calculates the cross-correlations between x_t and lagged values of y_t:

$$r_{xy}(l) = \frac{\sum_{t=1}^{n-l}(x_t - \bar{x})(y_{t+l} - \bar{y})}{ns_x s_y}, \quad l = 0, 1, \ldots, L$$

where

$$\bar{x} = \frac{\sum_{t=1}^{n} x_t}{n}$$

$$s_x{}^2 = \frac{\sum_{t=1}^{n}(x_t - \bar{x})^2}{n}$$

and similarly for y.

The ratio of standard deviations s_y/s_x is also returned, and a portmanteau statistic is calculated:

$$\textbf{stat} = n\sum_{l=1}^{L} r_{xy}(l)^2.$$

Provided n is large, L much less than n, and both x_t, y_t are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross-correlations between the series are zero, **stat** has a χ^2 distribution with L degrees of freedom. Values of **stat** in the upper tail of this distribution provide evidence against the null hypothesis.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5 Parameters

1: **x**[**nxy**] – const double *Input*

On entry: the n values of the x series.

2: **y**[**nxy**] – const double *Input*

On entry: the n values of the y series.

3: **nxy** – Integer *Input*

On entry: the length of the time series, n.

Constraint: **nxy** ≥ 2.

4: **nl** – Integer *Input*

On entry: the maximum lag for calculating cross-correlations, L.

Constraint: $1 \leq$ **nl** $<$ **nxy**.

5: **s** – double * *Output*

On exit: the ratio of the standard deviation of the y series to the standard deviation of the x series, s_y/s_x.

6: **r0** – double * *Output*

On exit: the cross-correlation between the x and y series at lag zero.

7: **r**[**nl**] – double *Output*

On exit: **r**$[l-1]$ contains the cross-correlations between the x and y series at lags l, $r_{xy}(l)$, for $l = 1, 2, \ldots, L$.

8: **stat** – double * *Output*

On exit: the statistic for testing for absence of cross-correlation.

9: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **nl** $= \langle value \rangle$.
Constraint: **nl** > 1.

On entry, **nxy** $= \langle value \rangle$.
Constraint: **nxy** > 1.

NE_INT_2

On entry, **nl** $\geq$ **nxy**: **nl** $= \langle value \rangle$, **nxy** $= \langle value \rangle$.

NE_ZERO_VARIANCE

One or both of the x and y series have zero variance.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

All computations are believed to be stable.

8 Further Comments

The time taken by the routine is approximately proportional to nL.

9 Example

The example program reads two time series of length 20. It calculates and prints the cross-correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

9.1 Program Text

```
/* nag_tsa_cross_corr (g13bcc) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  double r0xy, r0yx, statxy, statyx, sxy, syx;
  Integer exit_status, i, nl, nxy;
  NagError fail;

  /* Arrays */
  double *rxy = 0, *ryx = 0, *x = 0, *y = 0;

  INIT_FAIL(fail);
  exit_status = 0;

  Vprintf("g13bcc Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  /* Read series length and number of lags */
  Vscanf("%ld%ld%*[^\n] ", &nxy, &nl);

      /* Allocate memory */
      if ( !(rxy = NAG_ALLOC(nl, double)) ||
           !(ryx = NAG_ALLOC(nl, double)) ||
           !(x = NAG_ALLOC(nxy, double)) ||
           !(y = NAG_ALLOC(nxy, double)) )
        {
          Vprintf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }

      /* Read series */
      for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &x[i-1]);
      Vscanf("%*[^\n] ");

      for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &y[i-1]);
      Vscanf("%*[^\n] ");

      /* Call routine to calculate cross correlations between X and Y */
      g13bcc(x, y, nxy, nl, &sxy, &r0xy, rxy, &statxy, &fail);
          if (fail.code != NE_NOERROR)
            {
```

```
              Vprintf("Error from g13bcc, 1st call.\n%s\n",   fail.message);
              exit_status = 1;
              goto END;

            }

      /* Call routine to calculate cross correlations between Y and X */
      g13bcc(y, x, nxy, nl, &syx, &r0yx, ryx, &statyx, &fail);
          if (fail.code != NE_NOERROR)
            {
              Vprintf("Error from g13bcc, 2nd call.\n%s\n",   fail.message);
              exit_status = 1;
              goto END;

            }

      Vprintf("\n");
      Vprintf("                            Between        Between\n");
      Vprintf("                            X and Y        Y and X\n");
      Vprintf("\n");
      Vprintf("Standard deviation ratio%10.4f%15.4f\n", sxy, syx);
      Vprintf("\n");
      Vprintf("Cross correlation at lag\n");
      Vprintf("                         0");
      Vprintf("%10.4f%15.4f\n", r0xy, r0yx);
      for (i = 1; i <= nl; ++i)
         Vprintf("                          %4ld%10.4f%15.4f\n", i, rxy[i-1], ryx[i-
1]);
      Vprintf("\n");
      Vprintf("Test statistic          %10.4f%15.4f\n", statxy, statyx);

 END:
  if (rxy) NAG_FREE(rxy);
  if (ryx) NAG_FREE(ryx);
  if (x) NAG_FREE(x);
  if (y) NAG_FREE(y);

  return exit_status;
}
```

9.2 Program Data

```
g13bcc Example Program Data
    20    15
  0.02  0.05  0.08  0.03 -0.05  0.11 -0.01 -0.08 -0.08 -0.11
 -0.18 -0.19 -0.09  0.03  0.10  0.15 -0.14  0.07  0.09  0.16
  3.18  3.21  3.26  3.25  3.08  3.01  3.06  3.17  3.12  3.04
  3.26  3.45  3.33  3.70  3.31  3.81  3.33  2.96  3.28  3.10
```

9.3 Program Results

```
g13bcc Example Program Results

                           Between        Between
                           X and Y        Y and X

Standard deviation ratio    2.0053         0.4987

Cross correlation at lag
                        0    0.0568         0.0568
                        1    0.0438        -0.0151
                        2   -0.3762         0.3955
                        3   -0.4864         0.3417
                        4   -0.6294         0.5486
                        5   -0.3871         0.2291
                        6   -0.1690         0.3190
                        7   -0.0678         0.1980
                        8    0.0962         0.0438
                        9    0.0788        -0.1428
                       10    0.2910        -0.1376
```

```
                      11    0.0950        -0.0387
                      12    0.0547        -0.0380
                      13    0.1855        -0.1551
                      14    0.0243        -0.1536
                      15    0.0034        -0.0696

Test statistic             22.1269        17.2917
```