

## nag\_tsa\_spectrum\_bivar (g13cdc)

### 1. Purpose

**nag\_tsa\_spectrum\_bivar (g13cdc)** calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

### 2. Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_spectrum_bivar(Integer nxy, NagMeanOrTrend mt_correction,
                             double pxy, Integer mw, Integer is, double pw, Integer l,
                             Integer kc, double x[], double y[], Complex **g, Integer *ng,
                             NagError *fail)
```

### 3. Description

The supplied time series may be mean and trend corrected and tapered as in the description of `nag_tsa_spectrum_univar (g13cbc)` before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^*(\omega) = \frac{1}{2\pi n} \left\{ \sum_{t=1}^n y_t \exp(i\omega t) \right\} \times \left\{ \sum_{t=1}^n x_t \exp(-i\omega t) \right\}$$

for frequency values  $\omega_j = \frac{2\pi j}{K}$ ,  $0 \leq \omega_j \leq \pi$ .

A correction is made for bias due to any tapering.

As in the description of `nag_tsa_spectrum_univar (g13cbc)` for univariate frequency window smoothing, the smoothed spectrum is returned at a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}, \quad l = 0, 1, \dots, [L/2]$$

where  $[ ]$  denotes the integer part.

Its real part or co-spectrum  $cf(\nu_l)$ , and imaginary part or quadrature spectrum  $qf(\nu_l)$  are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} \tilde{w}_k f_{xy}^*(\nu_l + \omega_k)$$

where the weights  $\tilde{w}_k$  are similar to the weights  $w_k$  defined for `nag_tsa_spectrum_univar (g13cbc)`, but allow for an implicit alignment shift  $S$  between the series:

$$\tilde{w}_k = w_k \exp(-2\pi i S k / L).$$

It is recommended that  $S$  is chosen as the lag  $k$  at which the cross covariances  $c_{xy}(k)$  peak, so as to minimize bias.

If no smoothing is required, the integer  $M$  which determines the frequency window width  $\frac{2\pi}{M}$ , should be set to  $n$ .

The bandwidth of the estimates will normally have been calculated in a previous call of `nag_tsa_spectrum_univar (g13cbc)` for estimating the univariate spectra of  $y_t$  and  $x_t$ .

### 4. Parameters

**nxy**

Input: the length of the time series  $x$  and  $y$ ,  $n$ .

Constraint: **nxy**  $\geq 1$ .

**mt\_correction**

Input: whether the data are to be initially mean or trend corrected.

**mt\_correction** = **Nag\_NoCorrection** for no correction, **mt\_correction** = **Nag\_Mean** for mean correction, **mt\_correction** = **Nag\_Trend** for trend correction.

Constraint: **mt\_correction** = **Nag\_NoCorrection**, **Nag\_Mean** or **Nag\_Trend**

**pxy**

Input: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.

A value of 0.0 implies no tapering.

Constraint:  $0.0 \leq \mathbf{pxy} \leq 1.0$ .

**mw**

Input: the frequency width,  $M$ , of the smoothing window as  $2\pi/M$ .

A value of  $n$  implies that no smoothing is to be carried out.

Constraint:  $1 \leq \mathbf{mw} \leq \mathbf{nxy}$ .

**is**

Input: the alignment shift,  $S$ , between the  $x$  and  $y$  series. If  $x$  leads  $y$ , the shift is positive.

Constraint:  $-1 < \mathbf{is} < 1$ .

**pw**

Input: the shape parameter,  $p$ , of the trapezium frequency window.

A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.

If **mw** = **nxy** (i.e., no smoothing is carried out) then **pw** is not used.

Constraint:  $0.0 \leq \mathbf{pw} \leq 1.0$  if **mw**  $\neq$  **nxy**.

**l**

Input: the frequency division,  $L$ , of smoothed cross spectral estimates as  $2\pi/L$ .

Constraint:  $1 \geq 1$ .

**l** must be a factor of **kc** (see below).

**kc**

Input: the order of the fast Fourier transform (FFT) used to calculate the spectral estimates. **kc** should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq 2n$ , provided  $m \leq 20$ .

Constraint: **kc**  $\geq 2 \times \mathbf{nxy}$ .

**kc** must be a multiple of **l**. The largest prime factor of **kc** must not exceed 19, and the total number of prime factors of **kc**, counting repetitions, must not exceed 20. These two restrictions are imposed by nag\_fft\_real (c06eac) and nag\_fft\_hermitian (c06ebc) which perform the FFT.

**x[kc]**

Input: the **nxy** data points of the  $x$  series.

**y[kc]**

Input: the **nxy** data points of the  $y$  series.

**g**

Output: the complex vector which contains the **ng** cross spectral estimates in elements **g**[0] to **g**[**ng**−1]. The  $y$  series leads the  $x$  series.

The memory for this vector is allocated internally. If no memory is allocated to **g** (e.g. when an input error is detected) then **g** will be NULL on return. If repeated calls to this function are required then **NAG\_FREE** should be used to free the memory in between calls.

**ng**

Output: the number of spectral estimates,  $[L/2] + 1$ , whose separate parts are held in **g**.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

### NE\_BAD\_PARAM

On entry, parameter **mt\_correction** had an illegal value.

### NE\_INT\_ARG\_LT

On entry, **nxy** must not be less than 1: **nxy** =  $\langle value \rangle$ .

On entry, **mw** must not be less than 1: **mw** =  $\langle value \rangle$ .

On entry, **l** must not be less than 1: **l** =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_LT

On entry, **pxy** must not be less than 0.0: **pxy** =  $\langle value \rangle$ .

On entry, **pw** must not be less than 0.0: **pw** =  $\langle value \rangle$ .

### NE\_REAL\_ARG\_GT

On entry, **pxy** must not be greater than 1.0: **pxy** =  $\langle value \rangle$ .

On entry, **pw** must not be greater than 1.0: **pw** =  $\langle value \rangle$ .

### NE\_2\_INT\_ARG\_GT

On entry, **mw** =  $\langle value \rangle$  while **nxy** =  $\langle value \rangle$ . These parameters must satisfy  $\mathbf{mw} \leq \mathbf{nxy}$ .

### NE\_2\_INT\_ARG\_CONS

On entry, **l** =  $\langle value \rangle$  while **is** =  $\langle value \rangle$ . These parameters must satisfy  $\|\mathbf{is}\| < \mathbf{l}$  when  $\mathbf{l} > 0$ .

On entry, **kc** =  $\langle value \rangle$  while **nxy** =  $\langle value \rangle$ . These parameters must satisfy  $\mathbf{kc} \geq 2 * \mathbf{nxy}$  when  $\mathbf{nxy} > 0$ .

On entry, **kc** =  $\langle value \rangle$  while **l** =  $\langle value \rangle$ . These parameters must satisfy  $\mathbf{kc} \% \mathbf{l} \neq 0$  when  $\mathbf{l} > 0$ .

### NE\_FACTOR\_GT

At least one of the prime factors of **kc** is greater than 19.

### NE\_TOO\_MANY\_FACTORS

**kc** has more than 20 prime factors.

### NE\_ALLOC\_FAIL

Memory allocation failed.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 6. Further Comments

`nag_tsa_spectrum_bivar` carries out an FFT of length **kc** to calculate the sample cross spectrum. The time taken by the routine for this is approximately proportional to  $\mathbf{kc} \times \log(\mathbf{kc})$  (see `nag_fft_real` (c06eac) for further details).

### 6.1. Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

### 6.2. References

Bloomfield P (1976) *Fourier Analysis of Time Series: an Introduction*. Wiley.

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications*. Holden-Day.

## 7. See Also

None

## 8. Example

The example program reads 2 time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a  $2\pi/16$  frequency width of smoothing window, a window shape parameter of 0.5 and an alignment shift of 3. It then calls `nag_tsa_spectrum_bivar` to calculate the smoothed sample cross spectrum and prints the results.

## 8.1. Program Text

```

/* nag_tsa_spectrum_bivar(g13cdc) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagg13.h>

#define L 80
#define KC 8*L
#define NXYMAX 300

main()
{
    double x[KC], y[KC];
    double pxy;
    double pw;

    Complex *g;

    Integer i, j, ng, is;
    Integer mw;
    Integer nxy;
    Integer kc=KC, l=L;

    Vprintf("g13cdc Example Program Results\n");

    /*      Skip heading in data file */
    Vscanf("%*[^\\n] ");

    Vscanf("%ld ", &nxy);
    if (nxy > 0 && nxy <= NXYMAX)
    {
        for (i = 1; i <= nxy; ++i)
            Vscanf("%lf ", &x[i - 1]);
        for (i = 1; i <= nxy; ++i)
            Vscanf("%lf ", &y[i - 1]);

        /* Set parameters for call to g13cdc */
        /* Mean correction and 10 percent taper */
        pxy = 0.1;
        /* Window shape parameter and zero covariance at lag 16 */
        pw = .5;
        mw = 16;
        /* Alignment shift of 3 */
        is = 3;

        g13cdc(nxy, Nag_Mean, pxy, mw, is, pw, l, kc, x, y, &g,
            &ng, NAGERR_DEFAULT);

        Vprintf("\n          Returned sample spectrum\n");
        Vprintf("\n      Real   Imaginary      Real   Imaginary  \\\n");
        Vprintf("\n      part      part      part      part  \\\n");
        Vprintf("\n      part      part\n");
        for (j = 1; j <= ng; ++j)
            Vprintf("%5ld%8.4f%9.4f%s",
                j, a02bbc(g[j - 1]), a02bcc(g[j - 1]), (j%3==0 ? "\\n" : ""));
        Vprintf("\n");
        if (g)
            NAG_FREE(g);
    }
}

```

```

    }
    exit(EXIT_SUCCESS);
}

```

## 8.2. Program Data

g13cdc Example Program Data

```

296
-0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
 0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
 0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
 1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
 0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
 1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
 0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
 1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
 1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
 0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
 0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
 0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
 0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
 0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
 0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
 0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
 0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
 0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
 1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
 0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
 0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

### 8.3. Program Results

g13cdc Example Program Results

Returned sample spectrum

|    | Real<br>part | Imaginary<br>part |    | Real<br>part | Imaginary<br>part |    | Real<br>part | Imaginary<br>part |
|----|--------------|-------------------|----|--------------|-------------------|----|--------------|-------------------|
| 1  | -6.1563      | 0.0000            | 2  | -5.5905      | -2.0119           | 3  | -3.2711      | -2.7963           |
| 4  | -1.1803      | -2.3264           | 5  | -0.2061      | -1.8132           | 6  | 0.3434       | -1.1357           |
| 7  | 0.6200       | -0.7351           | 8  | 0.5967       | -0.3449           | 9  | 0.4523       | -0.0984           |
| 10 | 0.2391       | 0.0177            | 11 | 0.1129       | 0.0402            | 12 | 0.0564       | 0.0523            |
| 13 | 0.0134       | 0.0443            | 14 | -0.0032      | 0.0299            | 15 | -0.0057      | 0.0148            |
| 16 | -0.0057      | 0.0069            | 17 | -0.0033      | 0.0038            | 18 | -0.0011      | 0.0012            |
| 19 | -0.0004      | 0.0001            | 20 | -0.0004      | 0.0002            | 21 | -0.0003      | 0.0001            |
| 22 | -0.0001      | 0.0002            | 23 | -0.0002      | 0.0003            | 24 | -0.0002      | 0.0002            |
| 25 | -0.0002      | 0.0000            | 26 | -0.0004      | 0.0000            | 27 | -0.0002      | -0.0002           |
| 28 | -0.0001      | 0.0000            | 29 | -0.0001      | 0.0002            | 30 | -0.0001      | 0.0002            |
| 31 | -0.0002      | 0.0003            | 32 | -0.0002      | 0.0001            | 33 | -0.0001      | 0.0000            |
| 34 | 0.0000       | 0.0000            | 35 | 0.0000       | -0.0001           | 36 | 0.0001       | -0.0001           |
| 37 | 0.0001       | -0.0001           | 38 | 0.0001       | -0.0001           | 39 | 0.0000       | -0.0001           |
| 40 | 0.0000       | -0.0001           | 41 | 0.0001       | 0.0000            |    |              |                   |

---