NAG C Library Function Document

nag_shifted_log (s01bac)

1 Purpose

nag_shifted_log (s01bac) returns a value of the shifted logarithmic function, $\ln(1+x)$, via the function name.

2 Specification

```
double nag_shifted_log (double x, NagError *fail)
```

3 Description

nag_shifted_log (s01bac) computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The function is based on the Chebyshev expansion

$$\ln\frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4\sum_{k=0}^{\infty}\frac{p^{2k+1}}{2k+1}T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \dfrac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \dfrac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\dfrac{1}{\sqrt{2}} - 1, \sqrt{2} - 1\right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Parameters

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $\mathbf{x} > -1.0$.

2: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to -1.0: $\mathbf{x} = <value>$.

NE_BAD_PARAM

On entry, parameter <*value*> had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The returned result should be accurate almost to ***machine precision***, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in nag_shifted_log (s01bac).

8 Further Comments

Empirical tests show that the time taken for a call of nag_shifted_log (s01bac) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
/* nag_shifted_log (s01bac) Example Program
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s01bac Example Program Results\n");
  Vprintf("       x            y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s01bac(x, NAGERR_DEFAULT);
      Vprintf("%12.4e %12.4e\n", x, y);
    }
  return EXIT_SUCCESS;
}
```

9.2 Program Data

```
s01bac Example Program Data
  2.50e+0
  1.25e-1
 -9.06e-1
  1.29e-3
 -7.83e-6
  1.00e-9
```

9.3 Program Results

```
s01bac Example Program Results
       x            y
  2.5000e+00   1.2528e+00
  1.2500e-01   1.1778e-01
 -9.0600e-01  -2.3645e+00
  1.2900e-03   1.2892e-03
 -7.8300e-06  -7.8300e-06
  1.0000e-09   1.0000e-09
```