

nag_arctanh (s11aac)**1. Purpose**

nag_arctanh (s11aac) returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$.

2. Specification

```
#include <nag.h>
#include <nags.h>
```

```
double nag_arctanh(double x, NagError *fail)
```

3. Description

The function calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ the function is based on a Chebyshev expansion.

For $\frac{1}{2} < x^2 < 1$,

$$\operatorname{arctanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right).$$

4. Parameters

x

Input: the argument x of the function.

Constraint: $|x| < 1.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GE

On entry, $|x|$ must not be greater than or equal to 1.0: $x = \langle \text{value} \rangle$.

The function has been called with an argument greater than or equal to 1.0 in magnitude, for which $\operatorname{arctanh}$ is not defined. The result is returned as zero.

6. Further Comments**6.1. Accuracy**

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2) \operatorname{arctanh} x} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by at least a factor

$$\frac{x}{(1-x^2) \operatorname{arctanh} x}$$

in the result. The equality should hold if δ is greater than the **machine precision** (δ due to data errors etc.), but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However, in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where $\operatorname{arctanh}$ is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g. if x and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_arctanh(s11aac) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
    double x, y;

    Vprintf("s11aac Example Program Results\n");
    Vscanf("%*[^\\n]"); /* skip the first input line */
    Vprintf("      x      y\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s11aac(x, NAGERR_DEFAULT);
        Vprintf("%12.3e%12.3e\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s11aac Example Program Data
-0.5
0.0
0.5
-0.9999
```

8.3. Program Results

```
s11aac Example Program Results
      x      y
-5.000e-01 -5.493e-01
0.000e+00 0.000e+00
5.000e-01 5.493e-01
-9.999e-01 -4.952e+00
```
