nag_arcsinh (s11abc)

1. Purpose

nag_arcsinh (s11abc) returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_arcsinh(double x)
```

3. Description

The function calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ the function is based on a Chebyshev expansion.

For $|x| > 1$

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2+1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2+1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2+1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln |x|)$$

4. Parameters

x

Input: the argument x of the function.

5. Error Indications and Warnings

None.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2}\operatorname{arcsinh} x} \delta \right|.$$

That is, the relative error in the argument, x, is amplified by a factor at least

$$\frac{x}{\sqrt{1+x^2}\operatorname{arcsinh} x}$$

in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.), but if δ is simply due to round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation round-off.

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_arcsinh(s11abc) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double  x, y;

  Vprintf("s11abc Example Program Results\n");
  Vscanf("%*[^\n]s"); /* skip the first input line */
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s11abc(x);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s11abc Example Program Data
            -2.0
            -0.5
             1.0
             6.0
```

8.3. Program Results

```
s11abc Example Program Results
     x           y
  -2.000e+00  -1.444e+00
  -5.000e-01  -4.812e-01
   1.000e+00   8.814e-01
   6.000e+00   2.492e+00
```