nag_exp_integral (s13aac)

1. Purpose

nag_exp_integral (s13aac) returns the value of the exponential integral $E_1(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_exp_integral(double x, NagError *fail)
```

3. Description

The function evaluates

$$E_1(x) = \int_x^\infty \frac{e^{-u}}{u}\, du \qquad x > 0.$$

The approximation is based on several Chebyshev expansions.

4. Parameters

x

Input: the argument x of the function.
Constraint: $\mathbf{x} > 0.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LE

On entry, $\mathbf{x}$ must not be less than or equal to 0.0: $\mathbf{x} = \langle value \rangle$.
The function is not defined for this value and the result returned is zero.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in argument and result respectively, then in principle, $|\epsilon| \simeq |(e^{-x}/E_1(x))\,\delta|$, so the relative error in the argument is amplified in the result by at least a factor $e^{-x}/E_1(x)$. The equality should hold if δ is greater than the **machine precision** (δ due to data errors etc.), but if δ is simply a result of round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation and round-off.

It should be noted that, for small x, the amplification factor tends to zero and eventually the error in the result will be limited by **machine precision**.

For large x, $\epsilon \sim x\delta = \Delta$, the absolute error in the argument.

To guard against producing underflows, if x is larger than a machine-dependent value x_{hi}, the result is set directly to zero.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 5.1 p 228.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_exp_integral(s13aac) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s13aac Example Program Results\n");
  Vprintf("      x            y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s13aac(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s13aac Example Program Data
            2.0
            9.0
```

8.3. Program Results

```
s13aac Example Program Results
     x          y
  2.000e+00   4.890e-02
  9.000e+00   1.245e-05
```