NAG C Library Function Document

nag_complex_log_gamma (s14agc)

1 Purpose

nag_complex_log_gamma (s14agc) returns the value of the logarithm of the Gamma function $\ln\Gamma(z)$ for complex z, via the function name.

2 Specification

```
Complex nag_complex_log_gamma (Complex z, NagError *fail)
```

3 Description

nag_complex_log_gamma (s14agc) evaluates an approximation to the logarithm of the Gamma function $\ln\Gamma(z)$ defined for $\mathrm{Re}(z) > 0$ by

$$\ln\Gamma(z) = \ln\int_0^\infty e^{-t}t^{z-1}dt$$

where $z = x + iy$ is complex. It is extended to the rest of the complex plane by analytic continuation unless $y = 0$, in which case z is real and each of the points $z = 0, -1, -2, \ldots$ is a singularity and a branch point.

nag_complex_log_gamma (s14agc) is based on the method proposed by Kölbig (1972) in which the value of $\ln\Gamma(z)$ is computed in the different regions of the z plane by means of the formulae

$$\begin{aligned}
\ln\Gamma(z) &= (z-\tfrac{1}{2})\ \ln z - z + \tfrac{1}{2}\ \ln 2\pi + z\sum_{k=1}^{K}\frac{B_{2k}}{2k(2k-1)}z^{-2k} + R_K(z) && \text{if } x \geq x_0 \geq 0,\\
&= \ln\Gamma(z+n) - \ln\prod_{\nu=0}^{n-1}(z+\nu) && \text{if } x_0 > x \geq 0,\\
&= \ln\pi - \ln\Gamma(1-z) - \ln(\sin\ \pi z) && \text{if } x < 0,
\end{aligned}$$

where $n = [x_0] - [x]$, $\{B_{2k}\}$ are Bernoulli numbers (see Abramowitz and Stegun (1972)) and $[x]$ is the largest integer $\leq x$. Note that care is taken to ensure that the imaginary part is computed correctly, and not merely modulo 2π.

The function uses the values $K = 10$ and $x_0 = 7$. The remainder term $R_K(z)$ is discussed in Section 7.

To obtain the value of $\ln\Gamma(z)$ when z is real and positive, nag_log_gamma (s14abc) can be used.

4 References

Kölbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: **z** – Complex *Input*

On entry: the argument z of the function.

Constraint: $\mathrm{Re}(\mathbf{z})$ must not be 'too close' (see Section 6) to a non-positive integer when $\mathrm{Im}(\mathbf{z}) = 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_TOO_CLOSE_INTEGER

On entry, **z**.**re** is 'too close' to a non-positive integer when **z**.**im** $= 0.0$: **z**.**re** $= \langle value \rangle$, nint(**z**.**re**) $= \langle value \rangle$.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The remainder term $R_K(z)$ satisfies the following error bound:

$$\begin{aligned} |R_K(z)| &\leq \frac{|B_{2K}|}{|(2K-1)|} z^{1-2K} \\ &\leq \frac{|B_{2K}|}{|(2K-1)|} x^{1-2K} \text{ if } x \geq 0. \end{aligned}$$

Thus $|R_{10}(7)| < 2.5 \times 10^{-15}$ and hence in theory the function is capable of achieving an accuracy of approximately 15 significant digits.

8 Further Comments

None.

9 Example

The example program evaluates the logarithm of the Gamma function $\ln \Gamma(z)$ at $z = -1.5 + 2.5i$, and prints the results.

9.1 Program Text

```
/* nag_complex_log_gamma (s14agc) Example Program
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Complex y, z;
  Integer exit_status = EXIT_SUCCESS;
  NagError fail;

  INIT_FAIL(fail);
```

```
  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s14agc Example Program Results\n");
  Vprintf("     z                 ln(Gamma(z))\n");
  while (scanf(" (%lf,%lf)%*[^\n] ", &z.re, &z.im) != EOF)
    {
      y = s14agc(z, &fail);
      if (fail.code == NE_NOERROR)
        Vprintf("(%5.1f,%5.1f)   (%12.4e,%12.4e)\n", z.re, z.im, y.re, y.im);
      else
        {
          Vprintf("Error from s14agc.\n%s\n", fail.message);
          exit_status = 1;
          goto END;
        }
    }
 END:
  return exit_status;
}
```

9.2 Program Data

```
s14agc Example Program Data
 (-1.5, 2.5)  : Value of z
```

9.3 Program Results

```
s14agc Example Program Results
     z                 ln(Gamma(z))
( -1.5,  2.5)   ( -5.0140e+00, -4.0718e+00)
```