NAG C Library Function Document

nag_dawson (s15afc)

1 Purpose

nag_dawson (s15afc) returns a value for Dawson's Integral, $F(x)$, via the function name.

2 Specification

```
double nag_dawson (double x)
```

3 Description

nag_dawson (s15afc) evaluates an approximation for Dawson's Integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2}\, dt.$$

The function is based on two Chebyshev expansions:

For $0 < |x| \leq 4$,

$$F(x) = x {\sum_{r=0}}' a_r T_r(t), \quad \text{where} \quad t = 2\left(\frac{x}{4}\right)^2 - 1.$$

For $|x| > 4$,

$$F(x) = \frac{1}{x} {\sum_{r=0}}' b_r T_r(t), \quad \text{where} \quad t = 2\left(\frac{4}{x}\right)^2 - 1.$$

For $|x|$ near zero, $F(x) \simeq x$, and for $|x|$ large, $F(x) \simeq \frac{1}{2x}$. These approximations are used for those values of x for which the result is correct to ***machine precision***. For very large x on some machines, $F(x)$ may underflow and then the result is set exactly to zero (see the Users' Note for your implementation for details).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is considerably greater than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left|\frac{x(1-2xF(x))}{F(x)}\right|\delta.$$

The following graph shows the behaviour of the error amplification factor $\left|\frac{x(1-2xF(x))}{F(x)}\right|$:

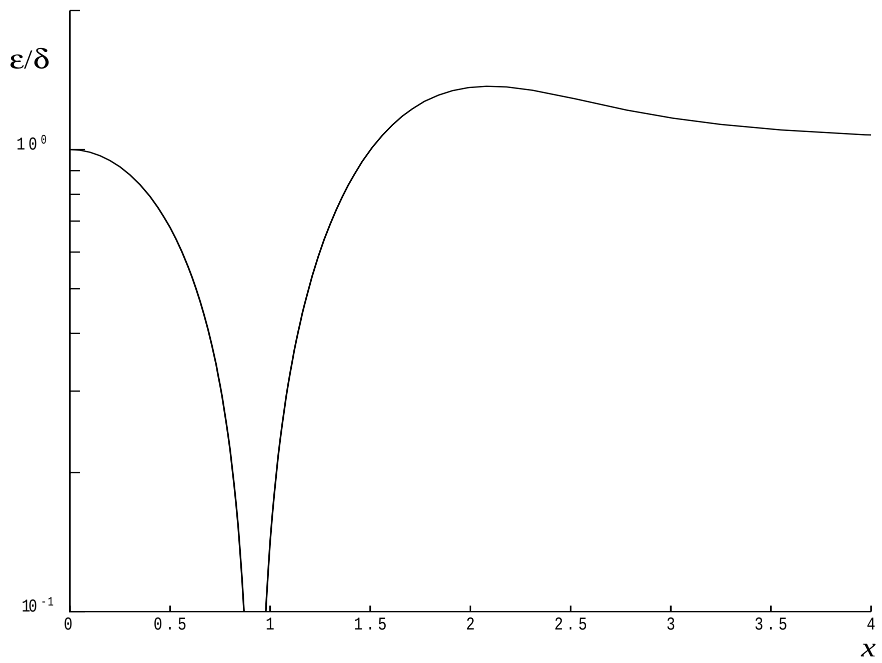

Figure 1

However if δ is of the same order as ***machine precision***, then rounding errors could make ϵ somewhat larger than the above relation indicates. In fact ϵ will be largely independent of x or δ, but will be of the order of a few times the ***machine precision***.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
/* nag_dawson (s15afc) Example Program
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  double x, y;

  Integer exit_status = EXIT_SUCCESS;
```

```
  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s15afc Example Program Results\n");
  Vprintf("       x            y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s15afc(x);
      Vprintf("%12.3e %12.3e\n", x, y);
    }
  return exit_status;
}
```

9.2 Program Data

```
s15afc Example Program Data
 -2.0
 -0.5
  1.0
  1.5
  2.0
  5.0
 10.0
```

9.3 Program Results

```
s15afc Example Program Results
       x            y
  -2.000e+00   -3.013e-01
  -5.000e-01   -4.244e-01
   1.000e+00    5.381e-01
   1.500e+00    4.282e-01
   2.000e+00    3.013e-01
   5.000e+00    1.021e-01
   1.000e+01    5.025e-02
```