

NAG C Library Function Document

nag_bessel_zeros (s17alc)

1 Purpose

nag_bessel_zeros (s17alc) determines the leading N zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$ for real x and non-negative α .

2 Specification

```
void nag_bessel_zeros (double a, Integer n, Integer mode, double rel,
                      double x[], NagError *fail)
```

3 Description

This routine attempts to find the leading N zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$, where x is real. When α is real, these functions each have an infinite number of real zeros, all of which are simple with the possible exception of $x = 0$. If $\alpha \geq 0$, the n th positive zero is denoted by $j_{\alpha,n}$, $j'_{\alpha,n}$, $y_{\alpha,n}$ and $y'_{\alpha,n}$, respectively, for $n = 1, 2, \dots, N$, except that $x = 0$ is counted as the first zero of $J'_\alpha(x)$ when $\alpha = 0$. Since $J'_0(x) = -J_1(x)$, it therefore follows that $j'_{0,1} = 0$ and $j'_{0,n} = -j_{1,n-1}$, for $n = 2, 3, \dots, N - 1$. Further details can be found in Abramowitz and Stegun (1972), 9.5.

nag_bessel_zeros is based on Algol 60 procedures given by Temme N M (1979). Initial approximations to the zeros are computed from asymptotic expansions. These are then improved by higher-order Newton iteration making use of the differential equation for the Bessel functions.

4 Parameters

- 1: **a** – double *Input*
On entry: the order α of the function.
Constraint: $0.0 \leq \mathbf{a} \leq 100000.0$.
- 2: **n** – Integer *Input*
On entry: the number N of zeros required.
Constraint: $\mathbf{n} \geq 1$.
- 3: **mode** – Integer *Input*
On entry: specifies the form of the function whose zeros are required as follows:
 - if **mode** = 1, then the zeros of $J_\alpha(x)$ are required;
 - if **mode** = 2, then the zeros of $Y_\alpha(x)$ are required;
 - if **mode** = 3, then the zeros of $J'_\alpha(x)$ are required;
 - if **mode** = 4, then the zeros of $Y'_\alpha(x)$ are required.*Constraint:* $1 \leq \mathbf{mode} \leq 4$.
- 4: **rel** – double *Input*
On entry: the relative accuracy to which the zeros are required.
Suggested value: the square root of the *machine precision*.
Constraint: $\mathbf{rel} > 0.0$.

- 5: **x[n]** – double Output
 On exit: the N required zeros of the function specified by **mode**.
- 6: **fail** – NagError * Input/Output
 The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_INT

On entry, **n** = *<value>*.
 Constraint: **n** $\geq$ 1.
 On entry, **mode** = *<value>*.
 Constraint: $1 \leq \mathbf{mode} \leq 4$.

NE_REAL

On entry, **a** = *<value>*.
 Constraint: $0.0 \leq \mathbf{a} \leq 100000.0$.
 On entry, **rel** = *<value>*.
 Constraint: **rel** $>$ 0.0.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

6.1 Accuracy

If the value of **rel** is set to 10^{-d} , then the required zeros should have approximately d correct significant digits.

6.2 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

Temme N M (1979) An algorithm with Algol 60 program for the computation of the *J. Comput. Phys.* **32** 270–279

Temme N M (1976) On the numerical evaluation of the ordinary Bessel function *J. Comput. Phys.* **21** 343–350

7 See Also

None.

8 Example

To determine the leading five positive zeros of the Bessel function $J_0(x)$.

8.1 Program Text

```

/* nag_bessel_zeros (s17alc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>
#include <nagx02.h>

int main(void)
{

#define NMAX 100

    double a, rel;
    double *x=0;
    Integer i;
    Integer exit_status=0;
    Integer mode;
    Integer n;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("s17alc Example Program Results\n\n");

    if (! (x = NAG_ALLOC(NMAX, double)))
    {
        Vprintf("Allocation failure\n");
        exit_status=-1;
    }
    /* Skip heading in data file */
    Vscanf("%*[^\n]");
    rel = sqrt (X02AJC);
    Vscanf("%lf %ld %ld", &a, &n, &mode);
    s17alc (a, n, mode, rel, x, &fail);

    if (fail.code == NE_NOERROR)
    {
        Vprintf("    a    n    mode            rel\n");
        Vprintf("                                (machine-dependent)\n\n");
        Vprintf(" %4.1f%3ld%6ld            %9.1e\n\n", a, n, mode, rel);
        if (mode == 1)
        {
            Vprintf("Leading N positive zeros of J\n");
        }
        else if (mode == 2)
        {
            Vprintf("Leading N positive zeros of Y\n");
        }
        else if (mode == 3)
        {

```

```

        if (a == 0.0)
        {
            Vprintf("Leading N non-negative zeros of J'\n");
        }
        else
        {
            Vprintf("Leading N positive zeros of J'\n");
        }
    }
    else if (mode == 4)
    {
        Vprintf("Leading N positive zeros of Y'\n\n");
    }
    for (i = 0; i <= n-1; ++i)
    {
        Vprintf("x = %12.4e\n", x[i]);
    }
    Vprintf("\n");
}
else
{
    Vprintf("Error from s17alc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
if (x) NAG_FREE (x);
return exit_status;
}

```

8.2 Program Data

s17alc Example Program Data
 0.0 5 1 : Values of a, n and mode

8.3 Program Results

s17alc Example Program Results

a	n	mode	rel (machine-dependent)
0.0	5	1	1.1e-08

Leading N positive zeros of J

x =	2.4048e+00
x =	5.5201e+00
x =	8.6537e+00
x =	1.1792e+01
x =	1.4931e+01
