nag_bessel_i0 (s18aec)

1. Purpose

nag_bessel_i0 (s18aec) returns the value of the modified Bessel function $I_0(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_bessel_i0(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the modified Bessel function of the first kind, $I_0(x)$.

The function is based on Chebyshev expansions.

For large x, the function must fail because of the danger of overflow in calculating e^x.

4. Parameters

x
Input: the argument x of the function.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT
On entry, |**x**| must not be greater than ⟨*value*⟩: **x** = ⟨*value*⟩.
|**x**| is too large and the function returns the approximate value of $I_0(x)$ at the nearest valid argument.

6. Further Comments

6.1. Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by $\epsilon \simeq |xI_1(x)/I_0(x)|\,\delta$.

However, if δ is of the same order as ***machine precision***, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x the amplification factor is approximately $x^2/2$, which implies strong attenuation of the error, but in general ϵ can never be less than the ***machine precision***.

For large x, $\epsilon \simeq x\delta$ and we have strong amplification of errors. However, the function must fail for quite moderate values of x, because $I_0(x)$ would overflow; hence in practice the loss of accuracy for large x is not excessive. Note that for large x the errors will be dominated by those of the **math library** function exp.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9 p 374.

7. See Also

nag_bessel_i1 (s18afc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_bessel_i0(s18aec) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s18aec Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s18aec(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s18aec Example Program Data
             0.0
             0.5
             1.0
             3.0
             6.0
             8.0
             10.0
             15.0
             20.0
             -1.0
```

8.3. Program Results

```
s18aec Example Program Results
     x           y
   0.000e+00   1.000e+00
   5.000e-01   1.063e+00
   1.000e+00   1.266e+00
   3.000e+00   4.881e+00
   6.000e+00   6.723e+01
   8.000e+00   4.276e+02
   1.000e+01   2.816e+03
   1.500e+01   3.396e+05
   2.000e+01   4.356e+07
  -1.000e+00   1.266e+00
```