nag_elliptic_integral_rc (s21bac)

1. Purpose

nag_elliptic_integral_rc (s21bac) returns a value of an elementary integral, which occurs as a degenerate case of an elliptic integral of the first kind,

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_elliptic_integral_rc(double x, double y, NagError *fail)
```

3. Description

This function calculates an approximate value for the integral

$$R_C(x,y) = \frac{1}{2}\int_0^\infty \frac{dt}{\sqrt{t+x}(t+y)}$$

where $x \geq 0$ and $y \neq 0$.

This function, which is related to the logarithm or inverse hyperbolic functions for $y < x$ and to inverse circular functions if $x < y$, arises as a degenerate form of the elliptic integral of the first kind. If $y < 0$, the result computed is the Cauchy principal value of the integral.

The basic algorithm, which is due to Carlson (1978) and Carlson (1988), is to reduce the arguments recursively towards their mean by the system:

$$\begin{array}{llll} x_0 & = x & y_0 & = y \\ \mu_n & = (x_n + 2y_n)/3 & S_n & = (y_n - x_n)/3\mu_n \\ \lambda_n & = y_n + 2\sqrt{x_n y_n} & & \\ x_{n+1} & = (x_n + \lambda_n)/4 & y_{n+1} & = (y_n + \lambda_n)/4. \end{array}$$

The quantity $|S_n|$ for $n = 0, 1, 2, 3, \ldots$ decreases with increasing n, eventually $|S_n| \sim 1/4^n$. For small enough S_n the required function value can be approximated by the first few terms of the Taylor series about the mean. That is

$$R_C(x,y) = \left(1 + \frac{3S_n^2}{10} + \frac{S_n^3}{7} + \frac{3S_n^4}{8} + \frac{9S_n^5}{22}\right) \Big/ \sqrt{\mu_n}.$$

The truncation error involved in using this approximation is bounded by $16|S_n|^6/(1-2|S_n|)$ and the recursive process is stopped when S_n is small enough for this truncation error to be negligible compared to the ***machine precision***.

Within the domain of definition, the function value is itself representable for all representable values of its arguments. However, for values of the arguments near the extremes the above algorithm must be modified so as to avoid causing underflows or overflows in intermediate steps. In extreme regions arguments are pre-scaled away from the extremes and compensating scaling of the result is done before returning to the calling program.

4. Parameters

x

y

Input: the arguments x and y of the function, respectively.
Constraint: $\mathbf{x} \geq 0.0$ and $\mathbf{y} \neq 0.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LT

On entry, **x** must not be less than 0.0: **x** = ⟨*value*⟩.
The function is undefined.

NE_REAL_ARG_EQ

On entry, **y** must not be equal to 0.0: **y** = ⟨*value*⟩.
The function is undefined and returns zero.

6. Further Comments

Symmetrised elliptic integrals returned by functions nag_elliptic_integral_rc, nag_elliptic_integral_rf (s21bbc), nag_elliptic_integral_rd (s21bcc) and nag_elliptic_integral_rj (s21bdc) can be related to the more traditional canonical forms (see Abramowitz and Stegun (1968)), as described in the Chapter Introduction.

6.1. Accuracy

In principle the function is capable of producing full ***machine precision***. However, round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 17.

Carlson B C (1978) *Computing Elliptic Integrals by Duplication* Department of Physics, Iowa State University (Preprint).

Carlson B C (1988) A Table of Elliptic Integrals of the Third Kind *Math. Comp.* **51** 267–280.

7. See Also

nag_elliptic_integral_rf (s21bbc)
nag_elliptic_integral_rd (s21bcc)
nag_elliptic_integral_rj (s21bdc)

8. Example

This example program simply generates a small set of non-extreme arguments which are used with the function to produce the table of low accuracy results.

8.1. Program Text

```
/* nag_elliptic_integral_rc(s21bac) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double rc, x, y;
  Integer ix;

  Vprintf("s21bac Example Program Results\n");
```

```
  Vprintf("     x      y       s21bac  \n");
  for (ix=1; ix<=3; ix++)
    {
      x = ix*0.5;
      y = 1.0;
      rc = s21bac(x, y, NAGERR_DEFAULT);
      Vprintf("%7.2f%7.2f%12.4f\n", x, y, rc);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

None.

8.3. Program Results

```
s21bac Example Program Results
     x      y       s21bac
   0.50   1.00      1.1107
   1.00   1.00      1.0000
   1.50   1.00      0.9312
```