LAPACK Working Note 29
On Global Combine Operations*

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Abstract

We discuss a hybrid strategy for implementing global combine operations on distributed memory MIMD multicomputers. A theoretical analysis is given and results from its implementation on the Intel iPSC/860 are reported.

1 Introduction

this paper, we address the implementation of the combine operation when vectors of data to
be combined are distributed among the processors (nodes) of a MIMD hy
Several solutions to this problem have appeared
that combines two of t
Ou

2 Assumptions

Target architectures for our algorithmare distributed memory hypercube mu
ing Multiple Instruction Multiple Data (MIMD) m
NCUBE2 and Transputer base
For our theoreti

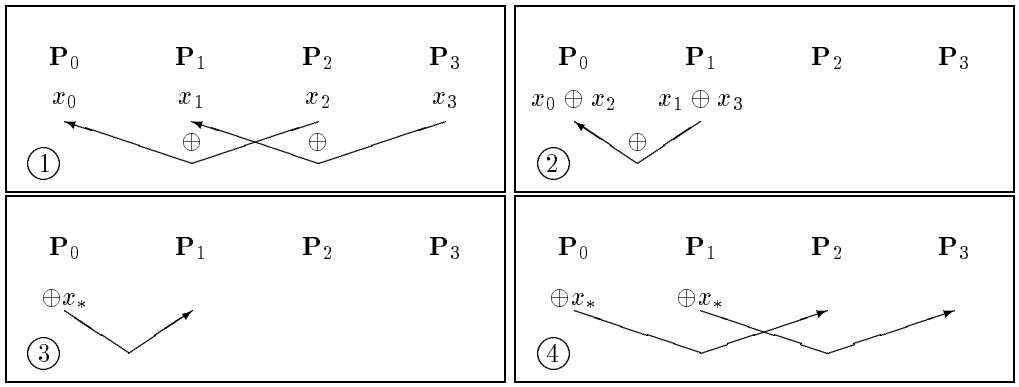

Figure 1: Version 1 on 4 nodes

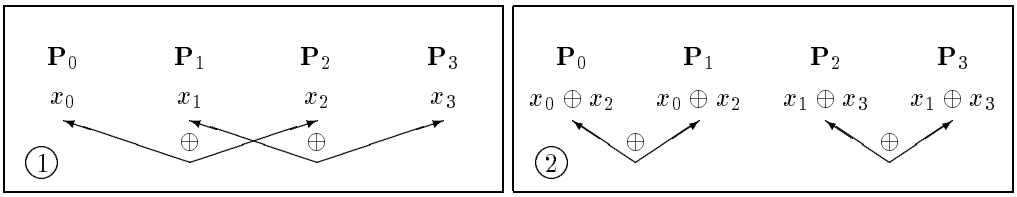

Figure 2: Version 2 on 4 nodes

3.2 Version 2

A second approach, described in [4], Section 14-5.4, starts by
contents of vector x_i with its neighbor indi
with the content

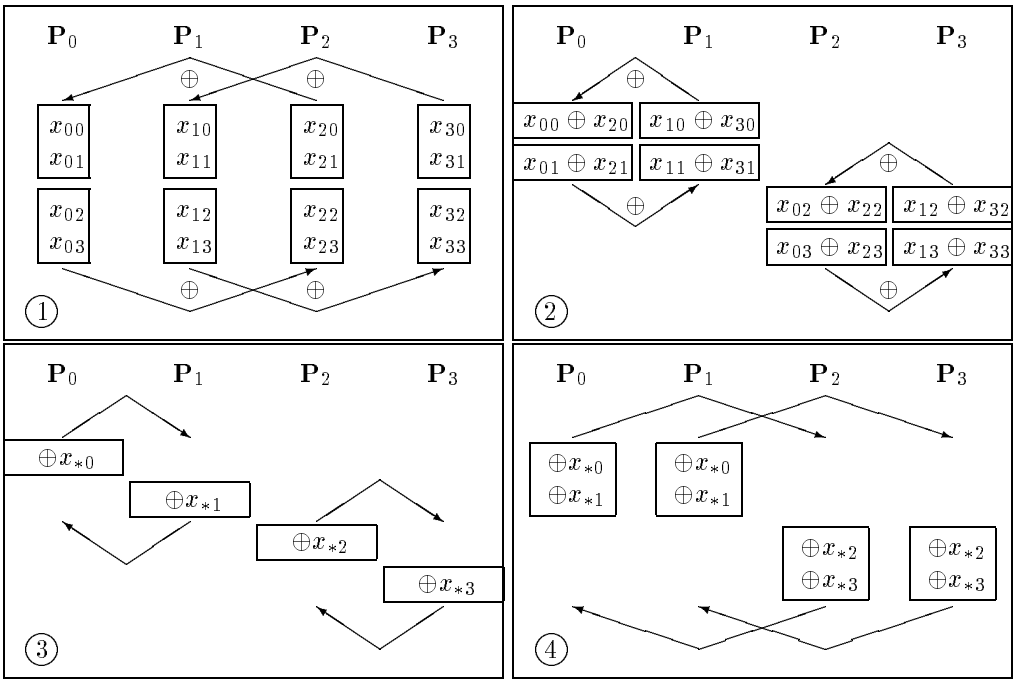

Figure 3: Version 3 on 4 nodes

not sent. This process proceeds for direction $d-2, \ldots, 0$, where the size o
communicated and combined is halved at each s
vector that results fr

combines of the partial result on subcubes of dimension $d-1$. This suggests a fa
strategies that combine Versions 2 and 3.
We can generate 2^d separate strategies by co
indicates that for the s

Proof: The first four results follow immediately from the definition of $T(S, n, d)$. Th
from the observation that the time of the first j steps of
time for the last $d-j$ steps. Ho
whe

Since $T(S, 2^{-(d-j-1)}n, j) > T(S, 2^{-(d-j)}n, j) \geq 0$ and $T(S, n, d) \leq T(R, n, d)$, we conclude th

$$\alpha + 2^{-(d-j-1)}n(\beta+\gamma) < 2\alpha + 2^{-(d-j)}n(2\beta+\gamma)$$

and hence

$$n < 2^{d-j}\frac{\alpha}{\gamma} < 2^{d-k}\frac{\alpha}{\gamma} \leq 2^{d-k}\frac{\alpha}{k(\beta+\gamma)+\gamma}$$

which contradicts the definition of k.

Case 2b: $j \leq k$. Then $S = (S_0, \ldots, S_k, 1, \ldots, 1)$ and

$$T(S, n, d) = \sum_{i=1}^{d-k-1} (2\alpha + 2^{-k}n(2\beta +$$

However,

$$T(\bar{S}, n, d) = \sum_{i=1}^{d-k-1} \left(2\alpha + 2^{-k}\right.$$

and, by Case 1 above,

$$T($$

which ag

```
hybridCOMB(n, x, y, d, S)
begin
     if S_{d-1}= 0 [a] then
          send (n, x, ngbr(i,d-1))
          recv (n, y, ngbr(i,d-1))
          combine (n, x, y)
          if d-1 > 0 call hybridCOMB(n, x, y, d-1, S)
     else
          let x0 = x[0,...,n/2-1], x1 = x[n/2,...,n]
          if bit(i,d-1)=0 then
               send(n/2, x1, ngbr(i,d-1))
               recv(n/2, y, ngbr(i,d-1))
               combine(n/2, x0, y)
               if d-1>0 then call hybridCOMB(n/2, x0, y, d-1, S)
               send(n/2, x0, ngbr(i,d-1))
               recv(n/2, x1, ngbr(i,d-1))
          else
               send(n/2, x0, ngbr(i,d-1))
               recv(n/2, y, ngbr(i,d-1))
               combine(n/2, x1, y)
               if d-1>0 call hybridCOMB(n/2, x1, y, d-1, S)
               send(n/2, x1, ngbr(i,d-1))
               recv(n/2, x0, ngbr(i,d-1))
end
```

[a] In Section 4 it will be shown that an optimal hybrid strategy can be obtained by deleting S from the calling sequence and replacing this condition by

$$n < 2\alpha/((d-1)(\beta+\gamma)+\gamma)$$

Figure 4: Hybrid global combine routine

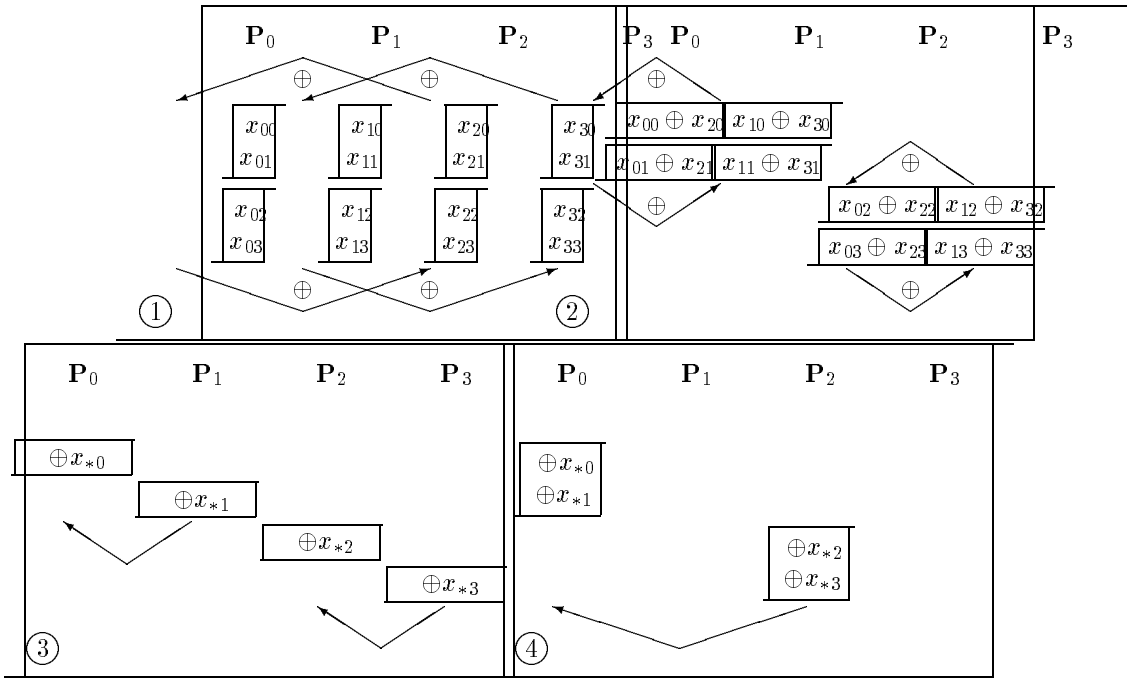

Figure 5: Second approach to combine-to-root on 4 nodes

5 Combine-to-Root

Some algorithms require the result of the global combine to only be known
loss of generality, we can take this node to be $\mathbf{P}_0$.
The first approach prog

```
hybridCOMB2R(n, x, y, d)
begin
     if n < 2α / ((d−1)(β+γ)+γ) then
          if bit(i,d-1)=1 then
               send (n, x, ngbr(i,d-1))
          else
               recv (n, y, ngbr(i,d-1))
               combine (n, x, y)
               if d-1 > 0 call hybridCOMB2R(n, x, y, d-1)
     else
          let x0 = x[0,...,n/2-1], x1 = x[n/2,...,n]
          if bit(i,d-1)=0 then
               send(n/2, x1, ngbr(i,d-1))
               recv(n/2, y, ngbr(i,d-1))
               combine(n/2, x0, y)
               if d-1>0 call hybridCOMB2R(n/2, x0, y, d-1)
               recv(n/2, x1, ngbr(i,d-1))
          else
               send(n/2, x0, ngbr(i,d-1))
               recv(n/2, y, ngbr(i,d-1))
               combine(n/2, x1, y)
               if d-1>0 call hybridCOMB2R(n/2, x1, y, d-1)
               send(n/2, x1, ngbr(i,d-1))
end
```

Figure 6: Optimal hybrid global combine-to-root routine

Figure 7: Predicted (*left*) and observed (*right*) time as a function of vector length n on 64 nodes when $\alpha = 525\mu$sec, $\beta = 2\mu$sec, and $\gamma = .35\mu$sec.

6 Experiments on the Intel iPSC/860

To test our theoretical results, we implemented the various global combine operations on t
Intel iPSC/860. Our experiments centered around a specific global combine operation, th
summation of single precision floating point vectors.

The Intel iPSC/860 is a commercial parallel processor that consists of
connected in a hypercube topology. Although this machine is somewhat
in Section 2, it can be programmed in such a way that all assumpti
In [3] it is shown that the cost for communicating a flo
iPSC/860 is roughly given by Assumption 5, with $\alpha = 136$
to single precision floating point numbers is $\gamma = .$
25 or less are communicated, the communica
this complication, we padded all communi
floating point numbers. There is so
general bookkeeping, yielding
overhead: $\beta = 2.0\mu$sec.
The first series of
proaches descr
communi
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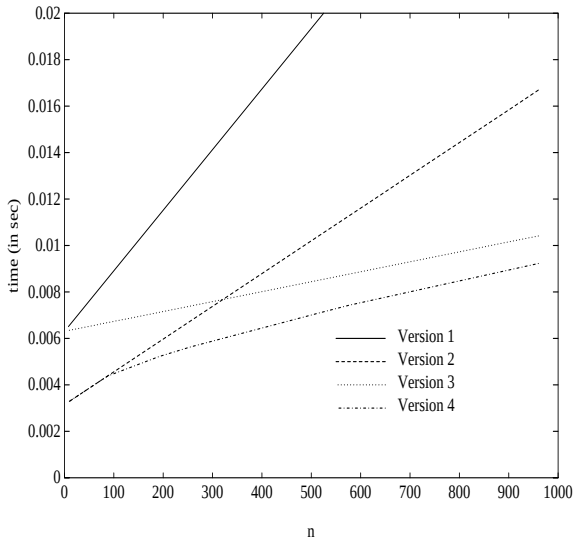
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
time (in sec)
0
100
200
300
400
500
600
700
800
900
1000
n
Version 1
Version 2
Version 3
Version 4

Figure 8: Observed time as a function of vector length n on 64 nodes. In this version, the length of vectors being communicated has not been adjusted, and the algorithm has been modified as described in Section 6, with $\alpha = 480\mu$sec, $\beta = 2\mu$sec, and $\gamma = .35\mu$sec.

communicated is less than 25. While there is a dramatic drop in execution time for Versio
2 when $n \leq 25$, Version 3 is particularly aided by the reduction in startup time, si
being communicated are halved at each step. For example, on a cube of dimens
vector is of length 512, the vectors communicated during the last tw
8, respectively, thereby reducing the execution time. This d
in Section 4. The following modification appears to
optimal choice as described in Section 4
each stage until the vector lengt
vectors thereafter, is us
method shows si

7 Conc

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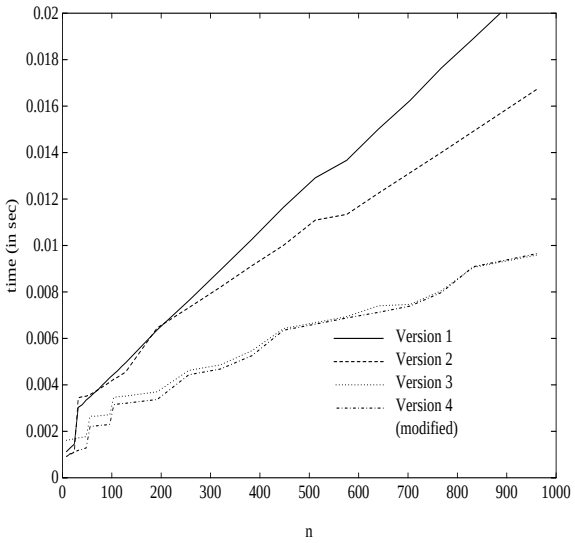
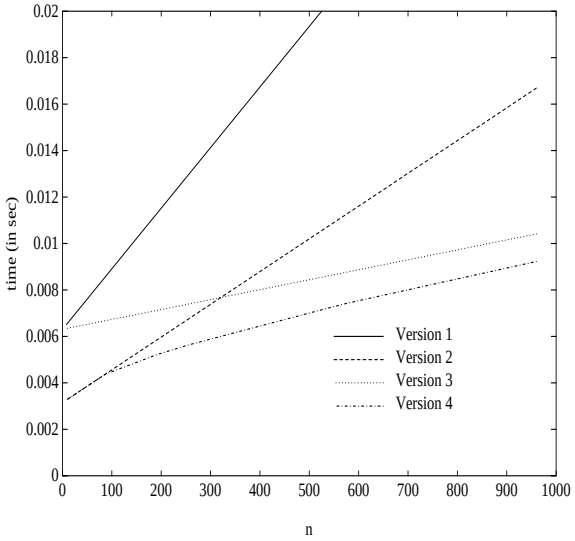
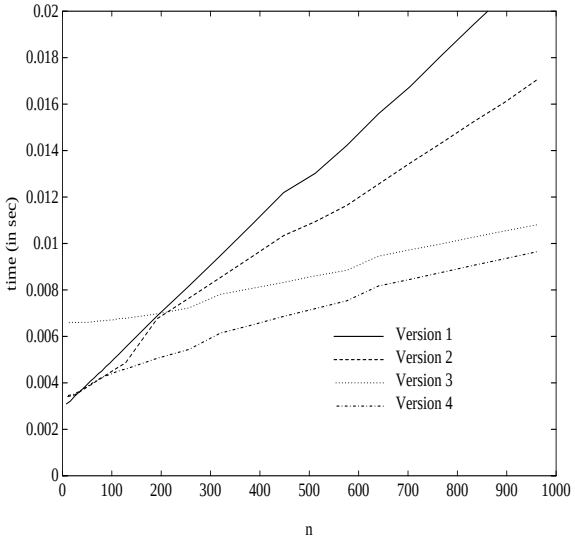

0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
time (in sec)
0
100
200
300
400
500
600
700
800
900
1000
n
Version 1
Version 2
Version 3
Version 4
(modified)


time (in sec)
n
Version 1
Version 2
Version 3
Version 4


time (in sec)
n
Version 1
Version 2
Version 3
Version 4

to be combined and communicated are partitioned into d (the dimension of the cube) equal parts,
where one part is exchanged in each direction. This kind of approach to increasing the utilization
of the communication network is discussed in [4], Chapter 21. The effect is to reduce the cons
multiplying the β term in the time complexities of Versions 2 and 3 by a factor d, w
carried through to reduce the execution time of the hybrid algorithmas well.
the scope of this paper.

Acknowledgements

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References

[1] E. Anderson, A. Benzoni, J. Dongarra, S. Moulton, S. O
R. van de Geijn. LAPACK for distributed memory arch
allel Processing for Scientific Computing, Fifth S

[2] J. J. Dongarra and R. A. van de Geijn. Reducti
on distributed memory architectures. LA
University of Tennessee, 1991.

[3] T. H. Dunigan. Performance of the
11491, Oak Ridge National Laborat

[4] G. Fox, M. Johnson, G. Lyzeng
Concurrent Processors, vol u

[5] C. Moler, J. Little, an