LAPACK Working Note 71
IBM RS/6000-550 & -590 Performance for Selected Routines in ESSL*/LAPACK /NAG†/IMSL‡§

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1 Overview

This document contains performance results for selected double-precision real and double-precision complex LAPACK routines and their equivalents in ESSL, NAG, and IMSL. The routines are listed in Figure 1.

2 Timing Environments

The experiments were conducted in two different timing environments: the IBM RS/6000-550 and the IBM RS/6000-590. In both cases the input matrices were identical for similar routines from different packages, thereby eliminating timing discrepancies resulting from pivoting, etc. Also, all timings were performed with no other users logged in to the machine.

2.1 IBM RS/6000 model 550

For the RS/6000 model 550 machine, routines were selected from the following four numerical algebra packages:

ESSL Version 2.2.1

*ESSL is a trademark of IBM.

†NAG is a trademark of The Numerical Algorithms Group Limited.

‡IMSL is a trademark of Visual Numerics, Inc.

§This work supported in part by the Office of Scientific Computing, U.S. Department of Energy, under Contract DE-AC05-84OR21400 and in part by IBM.

Figure 1: **The matched subroutines from each library and a brief description. Similar descriptions list the double-precision real routines before the double-precision complex.**

Description	LAPACK	ESSL	NAG	IMSL
General LU Factorization	DGETRF	DGEF	F07ADF	DLFTRG
General LU Factorization	ZGETRF	ZGEF	F07ARF	DLFTCG
Solve Ax = b Using LU	DGETRS	DGESM	F07AEF	DLFSRG
Solve Ax = b Using LU	ZGETRS	ZGESM	F07ASF	DLFSCG
Cholesky Factorization	DPOTRF	DPOF	F07FDF	DLFTDS
Cholesky Factorization	ZPOTRF	ZPOF	F07FRF	DLFTDH
Solve Ax = b Using Cholesky	DPOTRS	DPOSM	F07FEF	DLFSDS
Solve Ax = b Using Cholesky	ZPOTRS	ZPOSM	F07FSF	DLFSDH
Find Eigenvalues and Eigenvectors	DSPEV	DSLEV[a]	F02ABF	DEVSCF
Find Eigenvalues and Eigenvectors	ZHPEV	ZHLEV[b]	F02AXF	DEVSHF

[a] ESSL 2.2.1 accepts either DSPEV or DSLEV as the subroutine name for this routine; therefore, DSLEV has been used to prevent a name conflict with LAPACK's DSPEV. This is consistent throughout this report.

[b] ESSL 2.2.1 accepts either ZHPEV or ZHLEV as the subroutine name for this routine; therfore, ZHLEV has been used to prevent a name conflict with LAPACK's ZHPEV. This is consistent throughout this report.

LAPACK 1.1

NAG Mark 16

IMSL Version 2.0

All of the timings were made under the following conditions:

- AIX 3.2.5 was the operating system.
- XLF 3.1 was the Fortran compiler.
- *xlf -O2* `filename.f` was the compile command for **all** subroutines.
- *-lessl* was the ESSL library link for this architecture.
- The BLAS for all packages were supplied by the ESSL library. This gave all the packages an equal footing: that is to say, the algorithmic speed of different routines could be measured without an initial handicap of a slower or naive BLAS implementation.
- *READRTC* was the IBM-supplied utility subprogram for obtaining the timing data. It has a resolution of 1 microsecond.
- Blocksize selection routines were not modified for any of the blocking algorithms in any of the packages.

2.2 IBM RS/6000 model 590–256kByte data cache

For the IBM RS/6000 model 590 machine, routines were selected from the following two numerical algebra packages; IMSL and NAG have not yet been made available for the XLF 3.1 compiler on this architecture.

ESSL Version 2.2.1

LAPACK 1.1

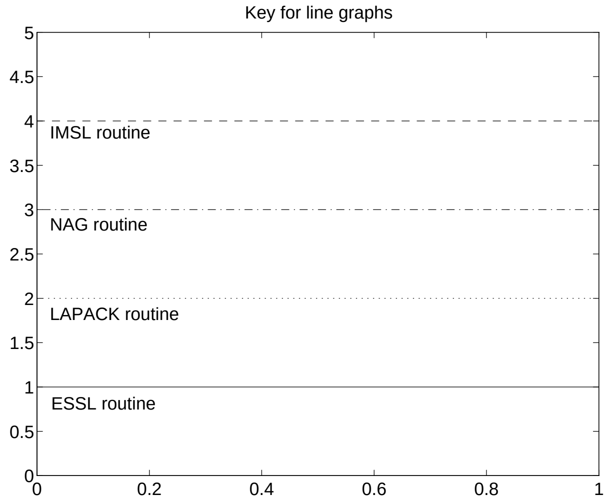
All of the timings were made under the following parameters:

- AIX 3.2.5 was the operating system.
- XLF 3.1 was the Fortran compiler.
- *xlf -O2 -qarch=pwrx* `filename.f` was the compile command for **all** subroutines.
- *-lesslp2* was the ESSL library link for this architecture.
- The BLAS were supplied by the ESSL package. This gave all the packages an equal footing: that is to say, the algorithmic speed of different routines could be measured without an initial handicap of a slower or naive BLAS implementation.
- *READRTC* was the IBM-supplied utility subprogram for obtaining the timing data. It has a resolution of 1 microsecond.
- Blocksize selection routines were not modified for any of the blocking algorithms in any of the packages.

3 Results

In this section, we discuss the computational results for each routine. Also discussed are the tests that were performed to ensure the validity of the timing data, as well as any specifics to the implementation of the subroutine The title of each subsection is the "4-tuple" of equivalent routines from ESSL, LAPACK, NAG, and IMSL, respectively.

All graphs utilize the following key:



Several points should be noted in reading the graphs:

- The data from both machines is graphed together; therefore, six lines appear on every graph (4 from the 550 and 2 from the 590). When two lines of the same type occur (only ESSL or LAPACK), the "faster" line always represents the IBM RS/6000-590. This format allows the comparison of all the numerical packages on the IBM RS/6000-550 as well as the contrast of ESSL and LAPACK on both machines.

- Some lines may overlap. In case of confusion, the timing data should be consulted.

3.1 *DGEF, DGETRF, F07ADF, DLFTRG*

The four routines DGEF, DGETRF, F07ADF, and DLFTRG compute the LU factorization of a general square DOUBLE PRECISION matrix. Note that NAG's implementation of F07ADF is LAPACK's DGETRF; consequently, the times are similar. Also, note that DGEF produces the same output format as DGETRF.

The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

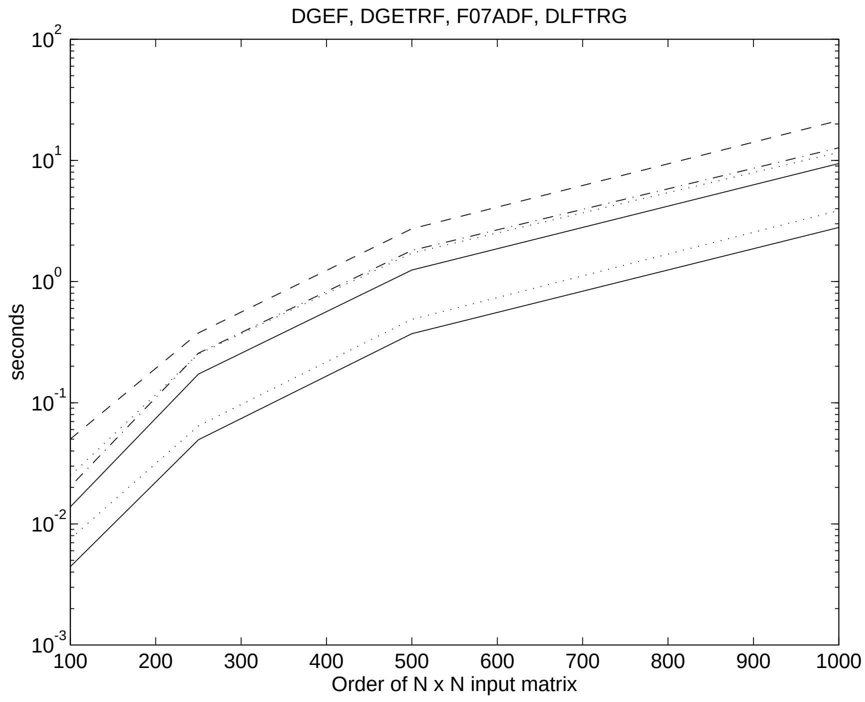
where ϵ is the machine precision.

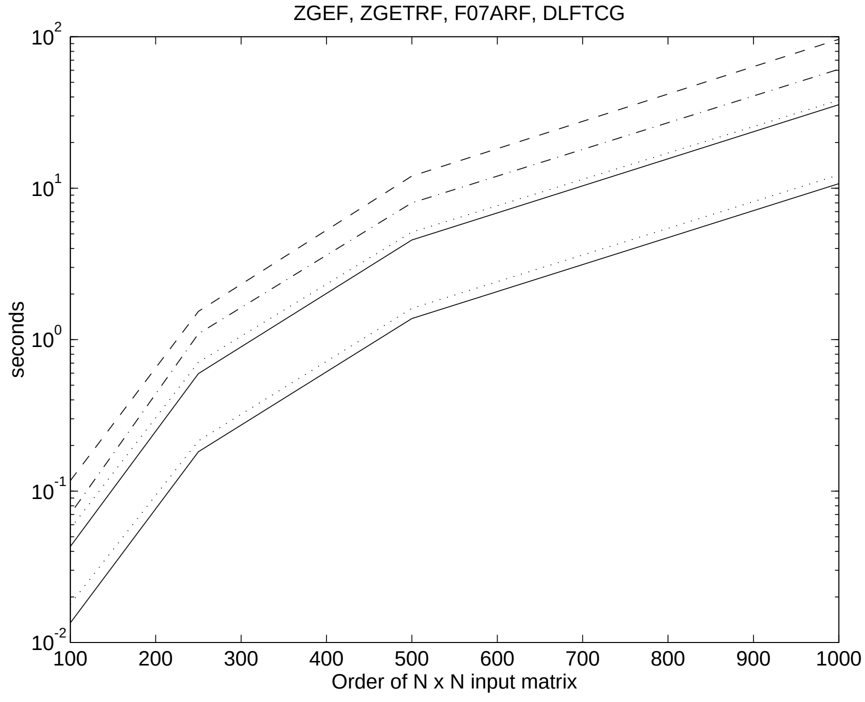
3.2 *ZGEF, ZGETRF, F07ARF, DLFTCG*

The routines ZGEF, ZGETRF, F07ARF, and DLFTCG compute the LU factorization of a general square COMPLEX*16 matrix. Note that NAG's implementation of F07ARF is LAPACK's ZGETRF; consequently, the times are similar. Also, note that ZGEF produces the same output format as ZGETRF. The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

where ϵ is the machine precision.


DGEF, DGETRF, F07ADF, DLFTRG
seconds
Order of N x N input matrix


ZGEF, ZGETRF, F07ARF, DLFTCG
seconds
Order of N x N input matrix

3.3 *DGESM, DGETRS, F07AE , DLFSRG*

The routines DGESM, DGETRS, F07AEF, and DLFSRG perform a DOUBLE PRECISION triangular solve, given a solution matrix and the factorization from Section 3.1. Note that NAG's implementation of F07AEF, ESSL's implementation of DGESM, and LAPACK's implementation of DGETRS are essentially the same; consequently, the times are similar. Also, IMSL's routine does not solve for multiple right-hand sides–rather, just a solution vector. Therefore, the following code was used to make IMSL's routine equivalent to the others:

```
      DO 100 IRHS = 1, NRHS
         CALL DLFSRG(..., B( 1, IRHS ), ...)
  100 CONTINUE
```

The following test was run to validate the results:

$$\frac{||Ax - b||}{||A|| ||x|| \epsilon}$$

where ϵ is the machine precision.

3.4 *ZGESM, ZGETRS, F07ASF, DLFSCG*

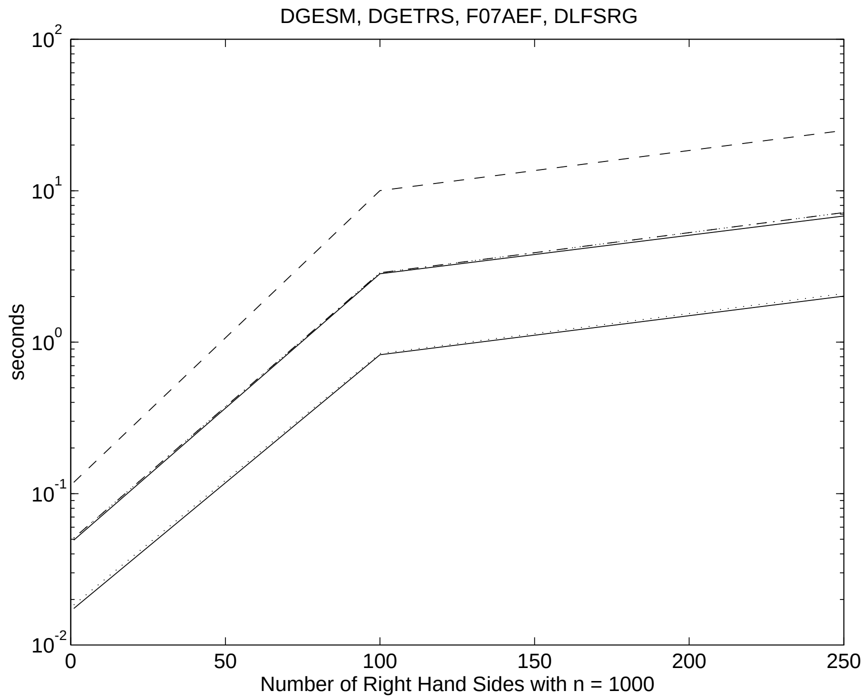
The routines ZGESM, ZGETRS, F07ASF, and DLFSCG perform a COMPLEX*16 triangular solve, given a solution matrix and the factorization from Section 3.2. Note that NAG's implementation of F07ASF, ESSL's implementation of ZGESM, and LAPACK's implementation of ZGETRS are essentially the same; consequently, the times are similar. Also, IMSL's routine does not solve for multiple right-hand sides–rather, just a solution vector. Therefore, the following code was used to make IMSL's routine equivalent to the others:

```
      DO 100 IRHS = 1, NRHS
         CALL DLFSCG(..., B( 1, IRHS ), ...)
  100 CONTINUE
```

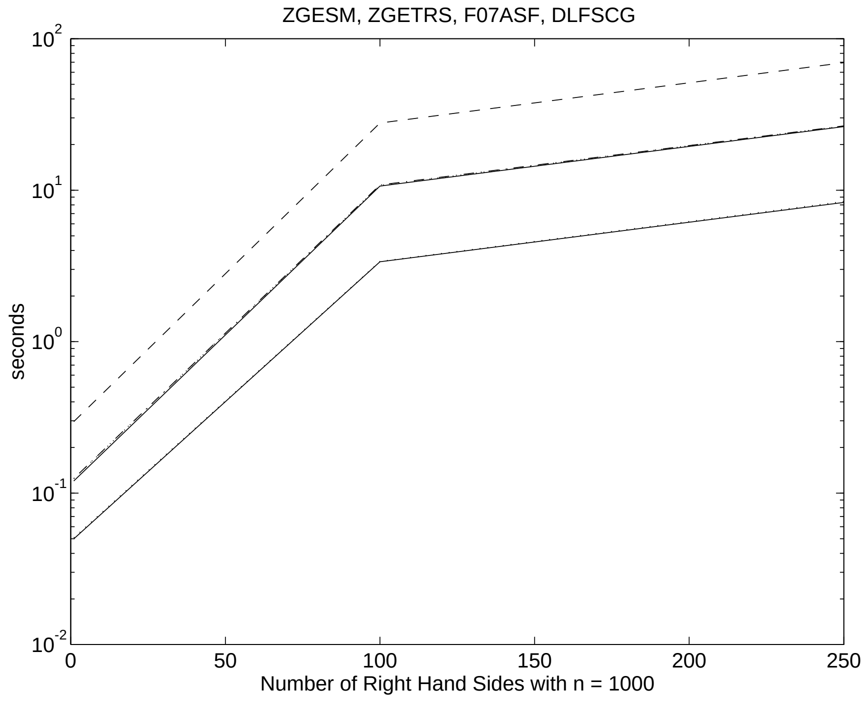
The following test was run to validate the results:

$$\frac{||Ax - b||}{||A|| ||x|| \epsilon}$$

where ϵ is the machine precision.



DGESM, DGETRS, F07AEF, DLFSRG
seconds
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0
50
100
150
200
250
Number of Right Hand Sides with n = 1000



ZGESM, ZGETRS, F07ASF, DLFSCG
seconds
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0
50
100
150
200
250
Number of Right Hand Sides with n = 1000

3.5 *DPOF, DPOTRF, F07FDF, DLFTDS*

The routines DPOF, DPOTRF, F07FDF, and DLFTDS compute the Cholesky factorization of a symmetric positive definite DOUBLE PRECISION matrix. Note that NAG's implementation of F07FDF is LAPACK's DPOTRF; consequently, the times are similar. For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

where ϵ is the machine precision.

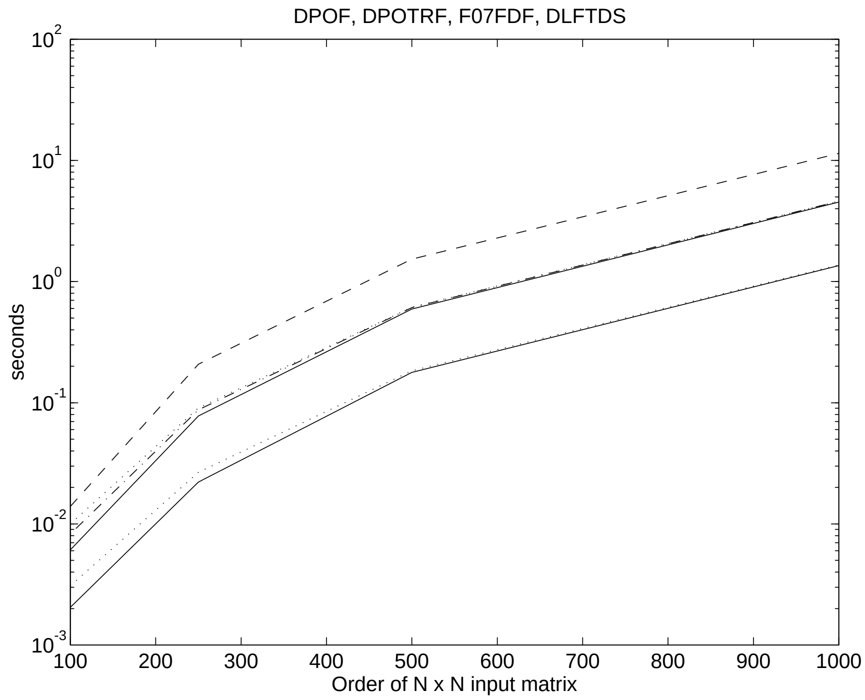
3.6 *ZPOF, ZPOTRF, F07FRF, DLFTDH*

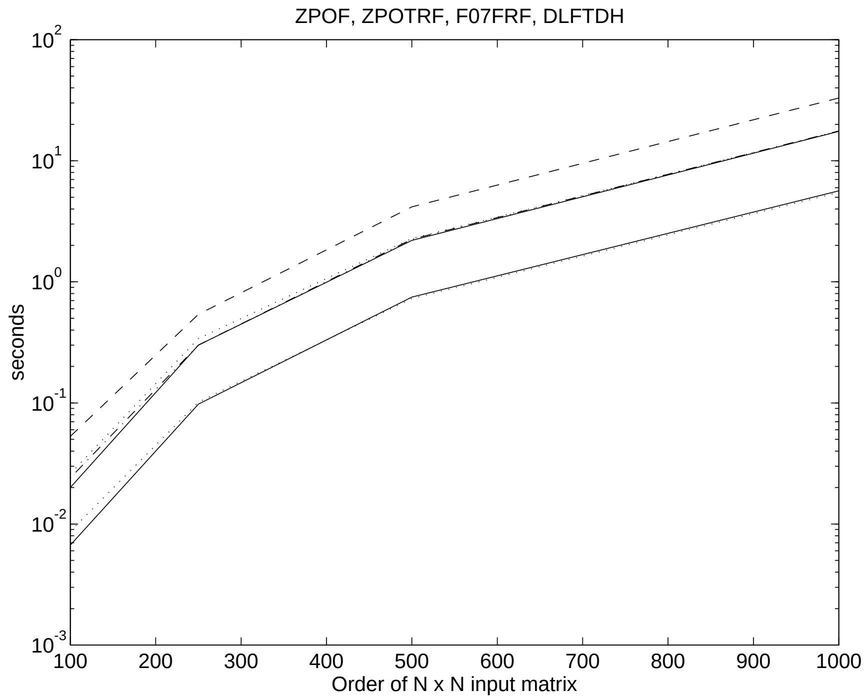
These routines compute the Cholesky factorization of a Hermitian positive definite COMPLEX*16 matrix. Note that NAG's implementation of F07FRF is LAPACK's ZPOTRF; consequently, the times are similar. For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

where ϵ is the machine precision.


DPOF, DPOTRF, F07FDF, DLFTDS
seconds
Order of N x N input matrix
10^2
10^1
10^0
10^-1
10^-2
10^-3
100
200
300
400
500
600
700
800
900
1000


ZPOF, ZPOTRF, F07FRF, DLFTDH
seconds
Order of N x N input matrix
10^2
10^1
10^0
10^-1
10^-2
10^-3
100
200
300
400
500
600
700
800
900
1000

3.7 *DPOSM, DPOTRS, F07FEF, DLFSDS*

The routines DPOSM, DPOTRS, F07FEF, and DLFSDS perform a DOUBLE PRECISION triangular solve, given a solution matrix and the factorization from Section 3.5. Note that NAG's implementation of F07FEF is LAPACK's DPOTRS; consequently, the times are similar. Also, IMSL's routine does not solve for multiple right-hand sides–rather, just a solution vector. Therefore, the following code was used to make IMSL's routine equivalent to the others:

```
          DO 100 IRHS = 1, NRHS
             CALL DLFSDS(..., B( 1, IRHS ), ...)
      100 CONTINUE
```

For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

where ϵ is the machine precision.

3.8 *ZPOSM, ZPOTRS, F07FSF, DLFSDH*

The routines ZPOSH, ZOTRS, F07FSF, and DLFSDH do a COMPLEX*16 triangular solve, given a solution matrix and the factorization from Section 3.6. Note that NAG's implementation of F07FSF is LAPACK's ZPOTRS; consequently, the times are similar. Also, IMSL's routine does not solve for multiple right-hand sides–rather, just a solution vector. Therefore, the following code was used to make IMSL's routine equivalent to the others:

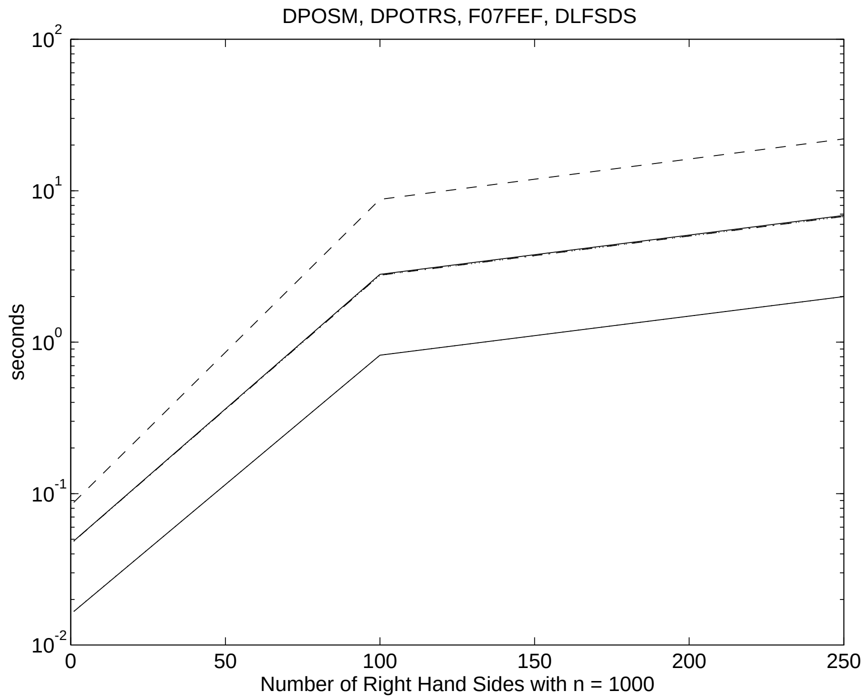
```
          DO 100 IRHS = 1, NRHS
             CALL DLFSDH(..., B( 1, IRHS ), ...)
      100 CONTINUE
```

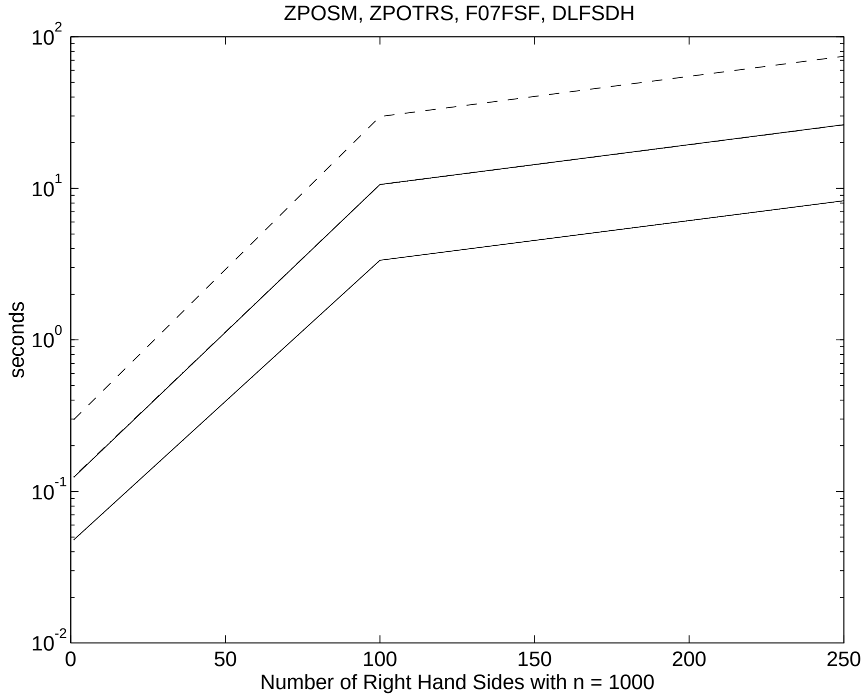
For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|Ax-b\|}{\|A\|\|x\|\epsilon}$$

where ϵ is the machine precision.


DPOSM, DPOTRS, F07FEF, DLFSDS
seconds
Number of Right Hand Sides with n = 1000
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0
50
100
150
200
250


ZPOSM, ZPOTRS, F07FSF, DLFSDH
seconds
Number of Right Hand Sides with n = 1000
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0
50
100
150
200
250

3.9 *DSLEV, DSPEV, F02ABF, DEVSCF*

The routines DSLEV, DSPEV, F02ABF, and DEVSCF compute the eigenvalues and eigenvectors of a DOUBLE PRECISION symmetric matrix. Unlike the first 8 routines, NAG's F02ABF is *not* equivalent to LAPACK's DSPEV. Also, LAPACK and ESSL use input matrices in *packed* form, whereas NAG and IMSL do not. For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|A - ZDZ^T\|}{\|A\| * n * \epsilon} \quad \text{and} \quad \frac{\|I - ZZ^T\|}{n * \epsilon}$$

where A is the symmetric input matrix of order n. Z contains the eigenvectors, and D contains the eigenvalues. I is the identity matrix, and ϵ is the machine precision.

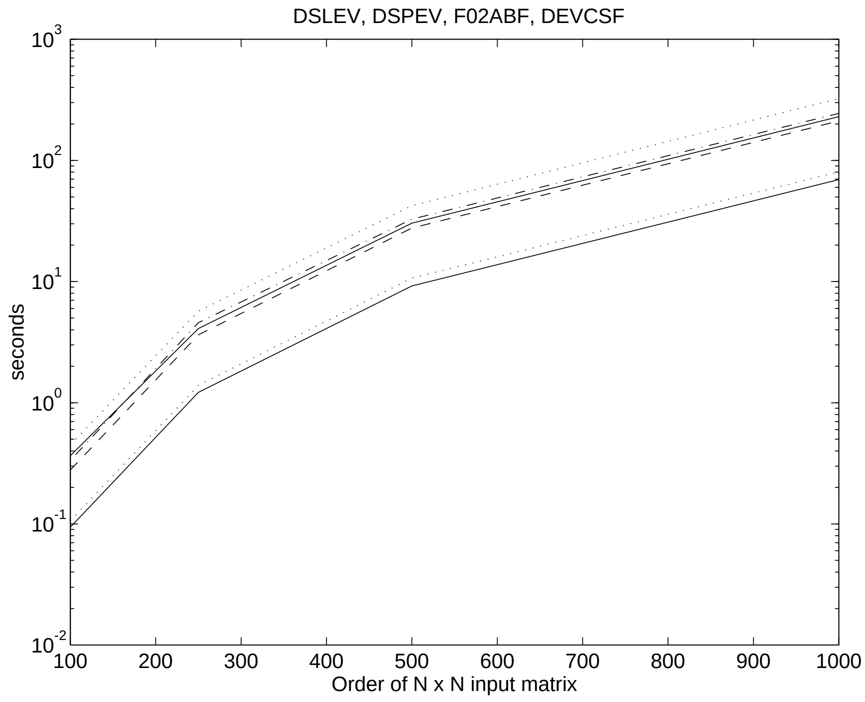
3.10 *ZHLEV, ZHPEV, F02AXF, DEVCHF*

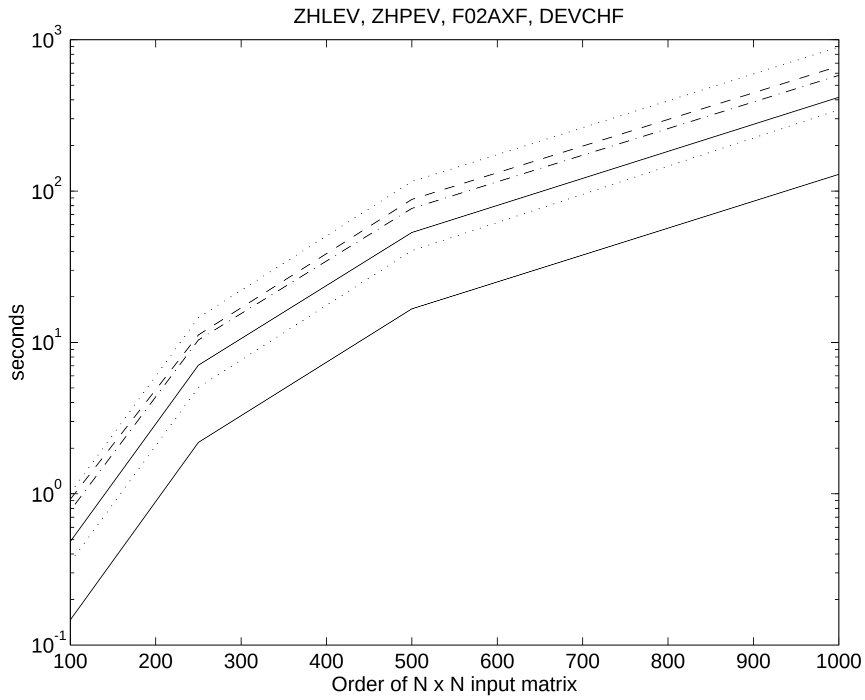
These routines compute the eigenvalues and eigenvectors of a COMPLEX*16 Hermitian matrix. Unlike the first 8 routines, NAG's F02AXF is NOT equivalent to LAPACK's ZHPEV. Also, LAPACK and ESSL use input matrices in *packed* form, whereas, NAG and IMSL do not. Further, NAG accepts its input matrix as two real matrices–one to hold the real values, and the other to hold the imaginary ones. ESSL, LAPACK, and IMSL accept one complex input matrix. For uniformity, the **upper** triangle is used for all timing data.

The following test was run to validate the results:

$$\frac{\|A - ZDZ^T\|}{\|A\| * n * \epsilon} \quad \text{and} \quad \frac{\|I - ZZ^T\|}{n * \epsilon}$$

where A is the symmetric input matrix of order n. Z contains the eigenvectors, and D contains the eigenvalues. I is the identity matrix, and ϵ is the machine precision.


DSLEV, DSPEV, F02ABF, DEVCSF
seconds
Order of N x N input matrix
100
200
300
400
500
600
700
800
900
1000
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


ZHLEV, ZHPEV, F02AXF, DEVCHF
seconds
Order of N x N input matrix
100
200
300
400
500
600
700
800
900
1000
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

4 Discussion

All of the tested subroutines appear to be numerically correct on both the IBM RS/6000-550 and the new IBM RS/6000-590. Further, the ESSL subroutines achieve the overall best performance of all the packages used. Also, as a side note, the ESSL BLAS have been tested and appear error-free; better yet, they are optimized for both architectures (as most manufacturer's BLAS packages typically are). Consequently, it is recommended that the ESSL BLAS library be linked in front of the numerical analysis package of choice.

A Timing Data

The following tables are output files of the times on the IBM RS/6000-550 and RS/6000-590, respectively. Counts for Megaflops are based upon the number of operations performed in the LAPACK routines.

A.1 550.out

```
Numerical packages used in this timing set:
    ESSL
   LAPACK
    NAG
    IMSL
The following parameter values will be used:
   N    :      100   250   500  1000
   NRHS :        1   100   250
Number of routine sets to be timed:  10

Start of Timings

*** Timing Set Separator ***

     100 x   100
 DGEF      .01381 seconds  47.9 Mflops
DGETRF     .02494 seconds  26.5 Mflops
F07ADF     .02035 seconds  32.5 Mflops
DLFTRG     .05011 seconds  13.2 Mflops

     250 x   250
 DGEF     .17203 seconds  60.4 Mflops
DGETRF    .25225 seconds  41.2 Mflops
F07ADF    .25609 seconds  40.6 Mflops
DLFTRG    .37602 seconds  27.6 Mflops

     500 x   500
 DGEF    1.24526 seconds  66.8 Mflops
DGETRF   1.71697 seconds  48.5 Mflops
F07ADF   1.80797 seconds  46.0 Mflops
DLFTRG   2.73070 seconds  30.5 Mflops

    1000 x  1000
 DGEF    9.42146 seconds  70.7 Mflops
DGETRF  11.64439 seconds  57.2 Mflops
F07ADF  12.71110 seconds  52.4 Mflops
DLFTRG  21.34845 seconds  31.2 Mflops

*** Timing Set Separator ***

     100 x   100
 ZGEF      .04310 seconds  61.6 Mflops
ZGETRF     .05652 seconds  47.0 Mflops
F07ARF     .07070 seconds  37.6 Mflops
DLFTCG     .11719 seconds  22.7 Mflops

     250 x   250
 ZGEF      .59706 seconds  69.7 Mflops
ZGETRF     .70884 seconds  58.7 Mflops
F07ARF    1.09264 seconds  38.1 Mflops
DLFTCG    1.53696 seconds  27.1 Mflops

     500 x   500
 ZGEF     4.55074 seconds  73.2 Mflops
ZGETRF    5.12570 seconds  65.0 Mflops
F07ARF    7.99196 seconds  41.7 Mflops
DLFTCG   12.03358 seconds  27.7 Mflops

    1000 x  1000
 ZGEF    35.56992 seconds  74.9 Mflops
ZGETRF   38.09008 seconds  70.0 Mflops
F07ARF   60.93470 seconds  43.7 Mflops
DLFTCG   96.09003 seconds  27.7 Mflops

*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
DGESM      .00098 seconds  20.3 Mflops
DGETRS     .00095 seconds  21.0 Mflops
F07AEF     .00093 seconds  21.4 Mflops
DLFSRG     .00151 seconds  13.1 Mflops
     Number of Right Hand Sides:   100
DGESM      .03243 seconds  61.4 Mflops
DGETRS     .03545 seconds  56.1 Mflops
F07AEF     .03366 seconds  59.1 Mflops
DLFSRG     .13329 seconds  14.9 Mflops
     Number of Right Hand Sides:   250
DGESM      .07936 seconds  62.7 Mflops
```

```
DGETRS    .10037 seconds  49.6 Mflops
F07AEF    .09990 seconds  49.8 Mflops
DLFSRG    .32836 seconds  15.2 Mflops

     250 x   250
     Number of Right Hand Sides:     1
DGESM     .00416 seconds  30.0 Mflops
DGETRS    .00454 seconds  27.5 Mflops
F07AEF    .00452 seconds  27.6 Mflops
DLFSRG    .00756 seconds  16.5 Mflops
     Number of Right Hand Sides:   100
DGESM     .18405 seconds  67.8 Mflops
DGETRS    .19275 seconds  64.7 Mflops
F07AEF    .20183 seconds  61.8 Mflops
DLFSRG    .72633 seconds  17.2 Mflops
     Number of Right Hand Sides:   250
DGESM     .44774 seconds  69.7 Mflops
DGETRS    .51719 seconds  60.3 Mflops
F07AEF    .51731 seconds  60.3 Mflops
DLFSRG   1.85189 seconds  16.8 Mflops

     500 x   500
     Number of Right Hand Sides:     1
DGESM     .01373 seconds  36.4 Mflops
DGETRS    .01504 seconds  33.2 Mflops
F07AEF    .01431 seconds  34.9 Mflops
DLFSRG    .02635 seconds  19.0 Mflops
     Number of Right Hand Sides:   100
DGESM     .70717 seconds  70.6 Mflops
DGETRS    .73682 seconds  67.8 Mflops
F07AEF    .73388 seconds  68.1 Mflops
DLFSRG   2.65112 seconds  18.8 Mflops
     Number of Right Hand Sides:   250
DGESM    1.72664 seconds  72.3 Mflops
DGETRS   1.88323 seconds  66.3 Mflops
F07AEF   1.88731 seconds  66.2 Mflops
DLFSRG   6.62045 seconds  18.9 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
DGESM     .04934 seconds  40.5 Mflops
DGETRS    .05089 seconds  39.3 Mflops
F07AEF    .05084 seconds  39.3 Mflops
DLFSRG    .11866 seconds  16.8 Mflops
     Number of Right Hand Sides:   100
DGESM    2.82658 seconds  70.7 Mflops
DGETRS   2.87006 seconds  69.7 Mflops
F07AEF   2.87292 seconds  69.6 Mflops
DLFSRG  10.00971 seconds  20.0 Mflops
     Number of Right Hand Sides:   250
DGESM    6.80839 seconds  73.4 Mflops
DGETRS   7.16865 seconds  69.7 Mflops
F07AEF   7.16805 seconds  69.7 Mflops
DLFSRG  24.98881 seconds  20.0 Mflops

*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
ZGESM     .00199 seconds  40.2 Mflops
ZGETRS    .00186 seconds  42.8 Mflops
F07ASF    .00185 seconds  43.1 Mflops
DLFSCG    .00360 seconds  22.2 Mflops
     Number of Right Hand Sides:   100
ZGESM     .11402 seconds  70.0 Mflops
ZGETRS    .11581 seconds  68.9 Mflops
F07ASF    .11489 seconds  69.5 Mflops
DLFSCG    .33419 seconds  23.9 Mflops
     Number of Right Hand Sides:   250
ZGESM     .27961 seconds  71.3 Mflops
ZGETRS    .30154 seconds  66.2 Mflops
F07ASF    .30331 seconds  65.8 Mflops
DLFSCG    .83361 seconds  23.9 Mflops

     250 x   250
     Number of Right Hand Sides:     1
ZGESM     .00908 seconds  55.0 Mflops
ZGETRS    .00934 seconds  53.5 Mflops
F07ASF    .00930 seconds  53.7 Mflops
DLFSCG    .01874 seconds  26.7 Mflops
     Number of Right Hand Sides:   100
ZGESM     .67880 seconds  73.6 Mflops
ZGETRS    .69327 seconds  72.0 Mflops
F07ASF    .69315 seconds  72.1 Mflops
DLFSCG   1.87768 seconds  26.6 Mflops
     Number of Right Hand Sides:   250
ZGESM    1.67864 seconds  74.4 Mflops
ZGETRS   1.75167 seconds  71.3 Mflops
```

```
F07ASF   1.75228 seconds  71.3 Mflops
DLFSCG   4.69669 seconds  26.6 Mflops

     500 x   500
     Number of Right Hand Sides:     1
ZGESM     .03174 seconds  63.0 Mflops
ZGETRS    .03260 seconds  61.3 Mflops
F07ASF    .03256 seconds  61.4 Mflops
DLFSCG    .07207 seconds  27.7 Mflops
     Number of Right Hand Sides:   100
ZGESM    2.68238 seconds  74.5 Mflops
ZGETRS   2.74301 seconds  72.9 Mflops
F07ASF   2.71448 seconds  73.6 Mflops
DLFSCG   7.15520 seconds  27.9 Mflops
     Number of Right Hand Sides:   250
ZGESM    6.60774 seconds  75.6 Mflops
ZGETRS   6.77147 seconds  73.8 Mflops
F07ASF   6.77807 seconds  73.7 Mflops
DLFSCG  17.91578 seconds  27.9 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
ZGESM     .12028 seconds  66.5 Mflops
ZGETRS    .12446 seconds  64.3 Mflops
F07ASF    .12465 seconds  64.2 Mflops
DLFSCG    .29612 seconds  27.0 Mflops
     Number of Right Hand Sides:   100
ZGESM   10.64363 seconds  75.1 Mflops
ZGETRS  10.75328 seconds  74.4 Mflops
F07ASF  10.82651 seconds  73.9 Mflops
DLFSCG  27.78095 seconds  28.8 Mflops
     Number of Right Hand Sides:   250
ZGESM   26.27937 seconds  76.1 Mflops
ZGETRS  26.60404 seconds  75.2 Mflops
F07ASF  26.60625 seconds  75.2 Mflops
DLFSCG  69.50199 seconds  28.8 Mflops

*** Timing Set Separator ***

     100 x   100
 DPOF     .00610 seconds  55.5 Mflops
DPOTRF    .01000 seconds  33.8 Mflops
F07FDF    .00825 seconds  41.0 Mflops
DLFTDS    .01395 seconds  24.2 Mflops

     250 x   250
 DPOF     .07776 seconds  67.4 Mflops
DPOTRF    .09048 seconds  57.9 Mflops
F07FDF    .08737 seconds  60.0 Mflops
DLFTDS    .20766 seconds  25.2 Mflops

     500 x   500
 DPOF     .59384 seconds  70.4 Mflops
DPOTRF    .61534 seconds  67.9 Mflops
F07FDF    .60912 seconds  68.6 Mflops
DLFTDS   1.53247 seconds  27.3 Mflops

    1000 x  1000
 DPOF    4.52868 seconds  73.7 Mflops
DPOTRF   4.61709 seconds  72.3 Mflops
F07FDF   4.60352 seconds  72.5 Mflops
DLFTDS  11.43321 seconds  29.2 Mflops

*** Timing Set Separator ***

     100 x   100
 ZPOF     .01926 seconds  70.8 Mflops
ZPOTRF    .02563 seconds  53.2 Mflops
F07FRF    .02449 seconds  55.7 Mflops
DLFTDH    .05344 seconds  25.5 Mflops

     250 x   250
 ZPOF     .28881 seconds  72.8 Mflops
ZPOTRF    .33835 seconds  62.1 Mflops
F07FRF    .30192 seconds  69.6 Mflops
DLFTDH    .54172 seconds  38.8 Mflops

     500 x   500
 ZPOF    2.22075 seconds  75.4 Mflops
ZPOTRF   2.28085 seconds  73.4 Mflops
F07FRF   2.24692 seconds  74.5 Mflops
DLFTDH   4.15906 seconds  40.3 Mflops

    1000 x  1000
 ZPOF   17.47949 seconds  76.5 Mflops
ZPOTRF  17.60942 seconds  75.9 Mflops
F07FRF  17.63009 seconds  75.8 Mflops
DLFTDH  33.04114 seconds  40.4 Mflops
```

```
*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
DPOSM     .00069 seconds  28.9 Mflops
DPOTRS    .00071 seconds  28.3 Mflops
F07FEF    .00067 seconds  29.7 Mflops
DLFSDS    .00118 seconds  17.0 Mflops
     Number of Right Hand Sides:   100
DPOSM     .03128 seconds  63.9 Mflops
DPOTRS    .03120 seconds  64.1 Mflops
F07FEF    .03115 seconds  64.2 Mflops
DLFSDS    .11143 seconds  17.9 Mflops
     Number of Right Hand Sides:   250
DPOSM     .07549 seconds  66.2 Mflops
DPOTRS    .07571 seconds  66.0 Mflops
F07FEF    .07523 seconds  66.5 Mflops
DLFSDS    .27968 seconds  17.9 Mflops

     250 x   250
     Number of Right Hand Sides:     1
DPOSM     .00380 seconds  32.9 Mflops
DPOTRS    .00394 seconds  31.7 Mflops
F07FEF    .00371 seconds  33.7 Mflops
DLFSDS    .00656 seconds  19.0 Mflops
     Number of Right Hand Sides:   100
DPOSM     .17960 seconds  69.6 Mflops
DPOTRS    .17742 seconds  70.5 Mflops
F07FEF    .17874 seconds  69.9 Mflops
DLFSDS    .65458 seconds  19.1 Mflops
     Number of Right Hand Sides:   250
DPOSM     .43529 seconds  71.8 Mflops
DPOTRS    .43542 seconds  71.8 Mflops
F07FEF    .43321 seconds  72.1 Mflops
DLFSDS   1.63897 seconds  19.1 Mflops

     500 x   500
     Number of Right Hand Sides:     1
DPOSM     .01336 seconds  37.4 Mflops
DPOTRS    .01334 seconds  37.5 Mflops
F07FEF    .01315 seconds  38.0 Mflops
DLFSDS    .02355 seconds  21.2 Mflops
     Number of Right Hand Sides:   100
DPOSM     .69936 seconds  71.5 Mflops
DPOTRS    .69738 seconds  71.7 Mflops
F07FEF    .69636 seconds  71.8 Mflops
DLFSDS   2.41227 seconds  20.7 Mflops
     Number of Right Hand Sides:   250
DPOSM    1.70554 seconds  73.3 Mflops
DPOTRS   1.70593 seconds  73.3 Mflops
F07FEF   1.70174 seconds  73.5 Mflops
DLFSDS   5.93529 seconds  21.1 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
DPOSM     .04870 seconds  41.1 Mflops
DPOTRS    .04843 seconds  41.3 Mflops
F07FEF    .04851 seconds  41.2 Mflops
DLFSDS    .08741 seconds  22.9 Mflops
     Number of Right Hand Sides:   100
DPOSM    2.80515 seconds  71.3 Mflops
DPOTRS   2.76205 seconds  72.4 Mflops
F07FEF   2.76806 seconds  72.3 Mflops
DLFSDS   8.77114 seconds  22.8 Mflops
     Number of Right Hand Sides:   250
DPOSM    6.87006 seconds  72.8 Mflops
DPOTRS   6.75116 seconds  74.1 Mflops
F07FEF   6.75768 seconds  74.0 Mflops
DLFSDS  21.99848 seconds  22.7 Mflops

*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
ZPOSM     .00171 seconds  47.1 Mflops
ZPOTRS    .00188 seconds  42.7 Mflops
F07FSF    .00167 seconds  48.2 Mflops
DLFSDH    .00381 seconds  21.1 Mflops
     Number of Right Hand Sides:   100
ZPOSM     .11168 seconds  72.0 Mflops
ZPOTRS    .11189 seconds  71.9 Mflops
F07FSF    .11170 seconds  72.0 Mflops
DLFSDH    .35981 seconds  22.3 Mflops
     Number of Right Hand Sides:   250
ZPOSM     .27350 seconds  73.5 Mflops
ZPOTRS    .27320 seconds  73.6 Mflops
F07FSF    .27353 seconds  73.5 Mflops
```

```
DLFSDH    .90033 seconds  22.3 Mflops

     250 x   250
     Number of Right Hand Sides:     1
ZPOSM     .01001 seconds  50.1 Mflops
ZPOTRS    .00903 seconds  55.5 Mflops
F07FSF    .00900 seconds  55.7 Mflops
DLFSDH    .02047 seconds  24.5 Mflops
     Number of Right Hand Sides:   100
ZPOSM     .67220 seconds  74.5 Mflops
ZPOTRS    .67288 seconds  74.5 Mflops
F07FSF    .67110 seconds  74.7 Mflops
DLFSDH   2.04851 seconds  24.5 Mflops
     Number of Right Hand Sides:   250
ZPOSM    1.65947 seconds  75.5 Mflops
ZPOTRS   1.66043 seconds  75.4 Mflops
F07FSF   1.65710 seconds  75.6 Mflops
DLFSDH   5.19694 seconds  24.1 Mflops

     500 x   500
     Number of Right Hand Sides:     1
ZPOSM     .03274 seconds  61.2 Mflops
ZPOTRS    .03290 seconds  60.9 Mflops
F07FSF    .03435 seconds  58.3 Mflops
DLFSDH    .07759 seconds  25.8 Mflops
     Number of Right Hand Sides:   100
ZPOSM    2.67634 seconds  74.8 Mflops
ZPOTRS   2.67118 seconds  74.9 Mflops
F07FSF   2.66576 seconds  75.1 Mflops
DLFSDH   8.20426 seconds  24.4 Mflops
     Number of Right Hand Sides:   250
ZPOSM    6.64089 seconds  75.4 Mflops
ZPOTRS   6.62695 seconds  75.5 Mflops
F07FSF   6.57015 seconds  76.2 Mflops
DLFSDH  19.32584 seconds  25.9 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
ZPOSM     .12407 seconds  64.5 Mflops
ZPOTRS    .12392 seconds  64.6 Mflops
F07FSF    .12514 seconds  64.0 Mflops
DLFSDH    .29777 seconds  26.9 Mflops
     Number of Right Hand Sides:   100
ZPOSM   10.60525 seconds  75.5 Mflops
ZPOTRS  10.60409 seconds  75.5 Mflops
F07FSF  10.59904 seconds  75.5 Mflops
DLFSDH  29.74984 seconds  26.9 Mflops
     Number of Right Hand Sides:   250
ZPOSM   26.29174 seconds  76.1 Mflops
ZPOTRS  26.22856 seconds  76.3 Mflops
F07FSF  26.25879 seconds  76.2 Mflops
DLFSDH  74.42543 seconds  26.9 Mflops

*** Timing Set Separator ***

     100 x   100
DSLEV     .36523 seconds  21.7 Mflops
DSPEV     .44989 seconds  17.6 Mflops
F02ABF    .33433 seconds  23.7 Mflops
DEVCSF    .27842 seconds  28.5 Mflops

     250 x   250
DSLEV    4.10146 seconds  28.7 Mflops
DSPEV    5.70087 seconds  20.6 Mflops
F02ABF   4.57444 seconds  25.7 Mflops
DEVCSF   3.63383 seconds  32.4 Mflops

     500 x   500
DSLEV   30.30769 seconds  30.1 Mflops
DSPEV   42.34420 seconds  21.5 Mflops
F02ABF  32.73366 seconds  27.9 Mflops
DEVCSF  27.66989 seconds  33.0 Mflops

    1000 x  1000
DSLEV  229.48141 seconds  31.1 Mflops
DSPEV  323.34836 seconds  22.1 Mflops
F02ABF 244.58197 seconds  29.2 Mflops
DEVCSF 211.76927 seconds  33.8 Mflops

*** Timing Set Separator ***

     100 x   100
ZHLEV     .48390 seconds  37.4 Mflops
ZHPEV    1.01247 seconds  17.9 Mflops
F02AXF    .77370 seconds  23.4 Mflops
DEVCHF    .91984 seconds  19.7 Mflops

     250 x   250
```

```
ZHLEV    7.06681 seconds  38.5 Mflops
ZHPEV   14.56624 seconds  18.7 Mflops
F02AXF  10.37188 seconds  26.3 Mflops
DEVCHF  11.19001 seconds  24.3 Mflops

     500 x   500
ZHLEV   53.18078 seconds  41.4 Mflops
ZHPEV  115.52283 seconds  19.1 Mflops
F02AXF  76.70922 seconds  28.7 Mflops
DEVCHF  87.96585 seconds  25.0 Mflops

    1000 x  1000
ZHLEV  415.99728 seconds  41.4 Mflops
ZHPEV  895.60843 seconds  19.2 Mflops
F02AXF 580.77693 seconds  29.7 Mflops
DEVCHF 668.79005 seconds  25.8 Mflops

End of Timings
```

A.2 590.out

```
 Numerical packages used in this timing set:
     ESSL
    LAPACK
 The following parameter values will be used:
    N    :      100   250   500  1000
    NRHS :        1   100   250
 Number of routine sets to be timed: 10

 Start of Timings

 *** Timing Set Separator ***

      100 x   100
  DGEF     .00444 seconds 148.9 Mflops
 DGETRF    .00753 seconds  87.9 Mflops

      250 x   250
  DGEF     .04951 seconds 209.7 Mflops
 DGETRF    .06439 seconds 161.3 Mflops

      500 x   500
  DGEF     .37139 seconds 224.0 Mflops
 DGETRF    .48771 seconds 170.6 Mflops

     1000 x  1000
  DGEF    2.79424 seconds 238.4 Mflops
 DGETRF   3.85433 seconds 172.8 Mflops

 *** Timing Set Separator ***

      100 x   100
  ZGEF     .01348 seconds 197.1 Mflops
 ZGETRF    .01794 seconds 148.1 Mflops

      250 x   250
  ZGEF     .18206 seconds 228.5 Mflops
 ZGETRF    .21470 seconds 193.8 Mflops

      500 x   500
  ZGEF    1.37634 seconds 242.0 Mflops
 ZGETRF   1.60849 seconds 207.1 Mflops

     1000 x  1000
  ZGEF   10.70659 seconds 249.0 Mflops
 ZGETRF  12.22530 seconds 218.0 Mflops

 *** Timing Set Separator ***

      100 x   100
      Number of Right Hand Sides:     1
 DGESM     .00034 seconds  58.0 Mflops
 DGETRS    .00032 seconds  61.6 Mflops
      Number of Right Hand Sides:   100
 DGESM     .00921 seconds 216.0 Mflops
 DGETRS    .00907 seconds 219.5 Mflops
      Number of Right Hand Sides:   250
 DGESM     .02303 seconds 216.0 Mflops
 DGETRS    .02276 seconds 218.6 Mflops

      250 x   250
      Number of Right Hand Sides:     1
 DGESM     .00130 seconds  95.9 Mflops
 DGETRS    .00163 seconds  76.6 Mflops
      Number of Right Hand Sides:   100
 DGESM     .05454 seconds 228.7 Mflops
 DGETRS    .05426 seconds 229.9 Mflops
      Number of Right Hand Sides:   250
 DGESM     .13310 seconds 234.3 Mflops
 DGETRS    .13962 seconds 223.4 Mflops

      500 x   500
      Number of Right Hand Sides:     1
 DGESM     .00505 seconds  98.9 Mflops
 DGETRS    .00535 seconds  93.4 Mflops
      Number of Right Hand Sides:   100
 DGESM     .20915 seconds 238.8 Mflops
 DGETRS    .21124 seconds 236.5 Mflops
      Number of Right Hand Sides:   250
 DGESM     .51023 seconds 244.7 Mflops
 DGETRS    .53649 seconds 232.8 Mflops

     1000 x  1000
      Number of Right Hand Sides:     1
 DGESM     .01743 seconds 114.7 Mflops
 DGETRS    .01841 seconds 108.6 Mflops
      Number of Right Hand Sides:   100
```

```
DGESM     .82599 seconds 242.0 Mflops
DGETRS    .84246 seconds 237.3 Mflops
     Number of Right Hand Sides:   250
DGESM    2.01335 seconds 248.2 Mflops
DGETRS   2.09311 seconds 238.8 Mflops

*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
ZGESM     .00075 seconds 105.8 Mflops
ZGETRS    .00075 seconds 106.0 Mflops
     Number of Right Hand Sides:   100
ZGESM     .04781 seconds 166.9 Mflops
ZGETRS    .04794 seconds 166.5 Mflops
     Number of Right Hand Sides:   250
ZGESM     .11915 seconds 167.4 Mflops
ZGETRS    .11995 seconds 166.3 Mflops

     250 x   250
     Number of Right Hand Sides:     1
ZGESM     .00368 seconds 135.7 Mflops
ZGETRS    .00389 seconds 128.3 Mflops
     Number of Right Hand Sides:   100
ZGESM     .24832 seconds 201.2 Mflops
ZGETRS    .24924 seconds 200.4 Mflops
     Number of Right Hand Sides:   250
ZGESM     .61376 seconds 203.5 Mflops
ZGETRS    .62532 seconds 199.7 Mflops

     500 x   500
     Number of Right Hand Sides:     1
ZGESM     .01323 seconds 151.1 Mflops
ZGETRS    .01370 seconds 145.9 Mflops
     Number of Right Hand Sides:   100
ZGESM     .89052 seconds 224.5 Mflops
ZGETRS    .94063 seconds 212.5 Mflops
     Number of Right Hand Sides:   250
ZGESM    2.20119 seconds 227.0 Mflops
ZGETRS   2.23438 seconds 223.7 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
ZGESM     .04983 seconds 160.5 Mflops
ZGETRS    .05070 seconds 157.7 Mflops
     Number of Right Hand Sides:   100
ZGESM    3.36467 seconds 237.7 Mflops
ZGETRS   3.38821 seconds 236.1 Mflops
     Number of Right Hand Sides:   250
ZGESM    8.30722 seconds 240.7 Mflops
ZGETRS   8.43493 seconds 237.1 Mflops

*** Timing Set Separator ***

     100 x   100
 DPOF     .00205 seconds 164.8 Mflops
DPOTRF    .00309 seconds 109.5 Mflops

     250 x   250
 DPOF     .02216 seconds 236.4 Mflops
DPOTRF    .02668 seconds 196.4 Mflops

     500 x   500
 DPOF     .17797 seconds 234.8 Mflops
DPOTRF    .18328 seconds 228.0 Mflops

    1000 x  1000
 DPOF    1.35571 seconds 246.2 Mflops
DPOTRF   1.37470 seconds 242.8 Mflops

*** Timing Set Separator ***

     100 x   100
 ZPOF     .00672 seconds 202.8 Mflops
ZPOTRF    .00871 seconds 156.5 Mflops

     250 x   250
 ZPOF     .09775 seconds 215.1 Mflops
ZPOTRF    .10162 seconds 206.9 Mflops

     500 x   500
 ZPOF     .74767 seconds 223.9 Mflops
ZPOTRF    .72744 seconds 230.1 Mflops

    1000 x  1000
 ZPOF    5.63974 seconds 236.9 Mflops
ZPOTRF   5.47636 seconds 244.0 Mflops
```

```
*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
DPOSM     .00023 seconds  85.9 Mflops
DPOTRS    .00023 seconds  85.6 Mflops
     Number of Right Hand Sides:   100
DPOSM     .00854 seconds 234.2 Mflops
DPOTRS    .00851 seconds 235.1 Mflops
     Number of Right Hand Sides:   250
DPOSM     .02121 seconds 235.8 Mflops
DPOTRS    .02139 seconds 233.8 Mflops

     250 x   250
     Number of Right Hand Sides:     1
DPOSM     .00126 seconds  99.1 Mflops
DPOTRS    .00125 seconds  99.8 Mflops
     Number of Right Hand Sides:   100
DPOSM     .05260 seconds 237.7 Mflops
DPOTRS    .05256 seconds 237.8 Mflops
     Number of Right Hand Sides:   250
DPOSM     .12891 seconds 242.4 Mflops
DPOTRS    .12849 seconds 243.2 Mflops

     500 x   500
     Number of Right Hand Sides:     1
DPOSM     .00469 seconds 106.6 Mflops
DPOTRS    .00460 seconds 108.8 Mflops
     Number of Right Hand Sides:   100
DPOSM     .20583 seconds 242.9 Mflops
DPOTRS    .20610 seconds 242.6 Mflops
     Number of Right Hand Sides:   250
DPOSM     .50275 seconds 248.6 Mflops
DPOTRS    .50278 seconds 248.6 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
DPOSM     .01666 seconds 120.0 Mflops
DPOTRS    .01669 seconds 119.8 Mflops
     Number of Right Hand Sides:   100
DPOSM     .82002 seconds 243.9 Mflops
DPOTRS    .82039 seconds 243.8 Mflops
     Number of Right Hand Sides:   250
DPOSM    2.00066 seconds 249.9 Mflops
DPOTRS   2.00077 seconds 249.9 Mflops

*** Timing Set Separator ***

     100 x   100
     Number of Right Hand Sides:     1
ZPOSM     .00068 seconds 118.9 Mflops
ZPOTRS    .00068 seconds 118.4 Mflops
     Number of Right Hand Sides:   100
ZPOSM     .04690 seconds 171.4 Mflops
ZPOTRS    .04699 seconds 171.1 Mflops
     Number of Right Hand Sides:   250
ZPOSM     .11626 seconds 172.9 Mflops
ZPOTRS    .11628 seconds 172.9 Mflops

     250 x   250
     Number of Right Hand Sides:     1
ZPOSM     .00350 seconds 143.3 Mflops
ZPOTRS    .00346 seconds 144.8 Mflops
     Number of Right Hand Sides:   100
ZPOSM     .24578 seconds 203.8 Mflops
ZPOTRS    .24638 seconds 203.3 Mflops
     Number of Right Hand Sides:   250
ZPOSM     .60861 seconds 205.8 Mflops
ZPOTRS    .60818 seconds 205.9 Mflops

     500 x   500
     Number of Right Hand Sides:     1
ZPOSM     .01284 seconds 155.9 Mflops
ZPOTRS    .01286 seconds 155.7 Mflops
     Number of Right Hand Sides:   100
ZPOSM     .88620 seconds 225.9 Mflops
ZPOTRS    .88635 seconds 225.9 Mflops
     Number of Right Hand Sides:   250
ZPOSM    2.18976 seconds 228.6 Mflops
ZPOTRS   2.19077 seconds 228.5 Mflops

    1000 x  1000
     Number of Right Hand Sides:     1
ZPOSM     .04790 seconds 167.1 Mflops
ZPOTRS    .04784 seconds 167.3 Mflops
     Number of Right Hand Sides:   100
ZPOSM    3.35386 seconds 238.7 Mflops
ZPOTRS   3.35393 seconds 238.6 Mflops
```

```
     Number of Right Hand Sides:   250
ZPOSM    8.27809 seconds 241.7 Mflops
ZPOTRS   8.28052 seconds 241.7 Mflops

*** Timing Set Separator ***

     100 x   100
DSLEV     .09377 seconds  84.7 Mflops
DSPEV     .10606 seconds  74.9 Mflops

     250 x   250
DSLEV    1.21790 seconds  96.7 Mflops
DSPEV    1.38857 seconds  84.8 Mflops

     500 x   500
DSLEV    9.18926 seconds  99.3 Mflops
DSPEV   10.68463 seconds  85.4 Mflops

    1000 x  1000
DSLEV   69.46343 seconds 102.9 Mflops
DSPEV   80.43700 seconds  88.9 Mflops

*** Timing Set Separator ***

     100 x   100
ZHLEV     .14683 seconds 123.2 Mflops
ZHPEV     .35243 seconds  51.3 Mflops

     250 x   250
ZHLEV    2.18221 seconds 124.8 Mflops
ZHPEV    5.04441 seconds  54.0 Mflops

     500 x   500
ZHLEV   16.64748 seconds 132.3 Mflops
ZHPEV   40.33818 seconds  54.6 Mflops

    1000 x  1000
ZHLEV  128.84442 seconds 133.8 Mflops
ZHPEV  343.85137 seconds  50.1 Mflops

End of Timings
```